Energy-Constrained Optimal Quantization for Wireless Sensor Networks†

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***Abstract*— As low power, low cost and longevity of transceivers are major requirements in wireless sensor networks, optimizing their design under energy constraints is of paramount importance. To this end, we develop quantizers under strict energy constraints to effect optimal reconstruction at the fusion center. Propagation, modulation, as well as transmitter and receiver structures are jointly accounted for using a binary symmetric channel model. We first optimize quantization for reconstructing a single sensor's measurement. Optimal number of quantization levels and optimal energy allocation across bits are derived. We then consider multiple sensors collaborating to estimate a deterministic parameter in noise. Similarly, optimum energy allocation and optimum number of quantization bits are derived analytically and tested with simulated examples.**

I. Introduction

Wireless sensor networks (WSN) are gaining increasing research interest for their emerging potential in both consumer and national security applications. Sensor networks can be deployed for surveillance, identification and tracking of targets. They can also serve as the first line of detection for various type of biological hazards, such as toxic gas attacks. In civilian applications, WSN can be used to monitor the environment and measure quantities such as temperature and pollution levels.

In most application scenarios, WSN nodes are powered by small batteries, which are practically non-rechargeable, either due to cost limitations or because they are deployed in hostile environments with high temperature, high pollution levels, or high nuclear radiation levels. This explains why energy-saving and energy-efficient WSN designs are of paramount importance. One approach to prolong the battery lifetime is the use of energy-harvesting radios as the ones in [1] with power dissipation levels below 100μW. A lot of research has been carried out to devise energy efficient algorithms in each layer of WSN [2]. Optimal modulation with minimum energy requirements to transmit a given number of bits with a prescribed bit error rate (BER) bound is considered in [3]. Energy efficient medium access control (MAC) and routing protocols are studied in [4] and [5], respectively.

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In this paper, we consider a WSN with a fusion center which collects data from the sensor nodes and performs the final information extraction task. A common goal in most WSN applications is to reconstruct the underlying physical phenomenon, e.g., temperature, based on sensor measurements. Energy as well as bandwidth limitations prevent sensor nodes from transmitting real valued (analog) data to the fusion center. This motivates the goal of this paper which is to derive optimal quantization schemes at sensor nodes under strict energy constraints. Optimality here is in the sense of minimizing a bound of the mean-absolute reconstruction error at the fusion center. The problem setup originates from the following considerations. Suppose we deploy a WSN powered by non-rechargeable batteries and expect it to operate a given number of times, which bounds the energy allowed per time of operation. One operation could be, for example, one time transmitting a burst of temperature measurements to the fusion center with bounded energy allowed per burst. The problem of designing quantizers to optimize pertinent reconstruction performance metrics under a given energy budget emerges naturally.

Related works include [6]–[10]. Assuming error-free transmission, [6] focuses on the impact of bandwidth constraints in WSN on the reconstruction performance. The effect of network MAC throughput on the distortion in reconstruction is considered in [7], which assumes real valued transmissions. Optimal quantization thresholds given the number of quantization levels and channel coding for binary symmetric channels (BSC) are jointly designed in [8] to minimize the mean square error of reconstruction. In [9], the scaling of reconstruction error with the number of quantization bits per Nyquist-period is studied, which reveals that low density quantization with high precision achieves reconstruction accuracy equivalent to dense quantization with low precision. The rate-distortion region when taking into account the possible failure of communication links and sensor nodes is presented in [10]. Different from these works, our objective is to optimize the quantization per node (including the number of quantization bits and the transmission energy allocation across bits) under a fixed total transmission energy per measurement in order to minimize the reconstruction error at the fusion center. This goal is distinct from existing ones in two directions: i) we account for the noisy channel between each sensor and

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the fusion center by modelling it as a BSC with crossover probability controlled by the transmitted bit energy; and ii) we allow different quantization bits to be allocated different energy and, thus, effect different crossover probabilities.

The rest of the paper is organized as follows. In Section II, we consider optimal quantization in a point-to-point (single-hop) link to recover a single sensor's measurement. In Section III, optimal quantization is addressed in a multi-sensor setting. Section IV provides some numerical results and Section V concludes the paper.

II. Point-To-Point Link

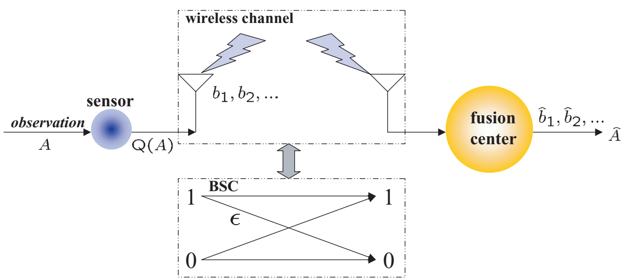

Fig. 1. System model of a single sensor's quantized measurement received through a BSC at the fusion center.

Let us consider the system depicted in Fig. 1, where a single sensor acquires a local measurement A, which is properly scaled so that $A \in [0, 1]$, and wishes to transmit it to the fusion center. For digital transmission, the sensor first quantizes the real valued measurement A to A_Q. Letting $A := \sum_{i=1}^{\infty} b_i 2^{-i}$, throughout this paper, we consider N-bit uniform quantization so that:

$$A_Q = \sum_{i=1}^{N} b_i 2^{-i}. \tag{1}$$

The quantization bits $\{b_i\}_{i=1}^{N}$ are transmitted through the wireless channel to the fusion center, and are received as $\{\hat{b}_i\}_{i=1}^{N}$. At the fusion center, the sensor's local measurement is reconstructed as:

$$\hat{A} = \sum_{i=1}^{N} \hat{b}_i 2^{-i}. \tag{2}$$

For simplicity, we consider only uncoded transmissions and channels that are memoryless with different bits experiencing independent fading effects. Under these conditions, we can model the wireless air-interface between the sensor and the fusion center as a binary symmetric channel (BSC) with crossover probability ϵ. In fact, the BSC model can be used to characterize a more general class of channels including multi-path fading and multi-access ones. Even for a channel with memory, BSC is still applicable provided that a suitable equalizer is incorporated, and $\{\hat{b}_i\}$ denote the bits at the output of the slicer that follows the equalizer.

Because one of the key issues in optimizing the design of sensor networks is the energy constraint, we are interested in the following problem:

If the allowable energy each time we transmit a measurement is fixed to $\mathcal{E}$, what is the optimal number of quantization bits and how can the energy per bit be allocated optimally in order to minimize the reconstruction error at the fusion center?

In the next two subsections, we will address this question.

A. Optimizing the Number of Quantization Bits

Let us consider a simple scenario where all quantization bits are allocated equal energy. We wish to find the optimal value of N in (1) which minimizes a meaningful metric of the reconstruction error. When using an N-bit quantizer, with the total transmission energy of all bits fixed to $\mathcal{E}$, the energy per bit depends clearly on N, since $\mathcal{E}_b = \mathcal{E}/N$. Noticing that the BSC model's crossover probability ϵ will generally be a function of the bit-energy-to-noise ratio, and letting N_0 denote the channel noise level, we can write ϵ as $\epsilon(\mathcal{E}_b/N_0)$ to make this functional relationship explicit. With $\mathcal{E}_b = \mathcal{E}/N$, we find that the crossover probability is actually a function of N with $\epsilon = \epsilon(\mathcal{E}/(NN_0))$.

The reconstruction error, which is defined to be $A - \hat{A}$, can be expressed as:

$$\begin{aligned} A - \hat{A} &= A - A_Q + A_Q - \hat{A} \\ &= \sum_{i=N+1}^{\infty} b_i 2^{-i} + \sum_{i=1}^{N} (b_i - \hat{b}_i) 2^{-i}. \end{aligned} \tag{3}$$

Using the triangle inequality, we can readily bound the absolute value of the reconstruction error as:

$$|A - \hat{A}| \le \sum_{i=N+1}^{\infty} 2^{-i} + \sum_{i=1}^{N} |b_i - \hat{b}_i| 2^{-i}. \tag{4}$$

Taking expectation on both sides of (4), we have:

$$\begin{aligned} \mathsf{E}[|A - \hat{A}|] &\le 2^{-N} + \sum_{i=1}^{N} \mathsf{E}[|b_i - \hat{b}_i|] 2^{-i} \\ &= 2^{-N} + \epsilon(\frac{\mathcal{E}}{NN_0}) \sum_{i=1}^{N} 2^{-i} \\ &= 2^{-N} + (1 - 2^{-N}) \epsilon(\frac{\mathcal{E}}{NN_0}) \\ &:= f(N). \end{aligned} \tag{5}$$

In order to minimize the mean-absolute reconstruction error, it suffices to minimize the bound $f(N)$ in (5) with respect to N. Under this criterion, the optimal number of quantization bits should be chosen as follows:

$$\begin{aligned} N_{opt} &= \arg\min_N f(N) \\ &= \arg\min_N \left[2^{-N} + (1 - 2^{-N}) \epsilon(\frac{\mathcal{E}}{NN_0}) \right]. \end{aligned} \tag{6}$$

With $\epsilon(\cdot)$ being a monotonically decreasing function of its argument, existence and uniqueness of N_{opt} in (6) are guaranteed. Although the minimization in (6) involves just a simple one-dimensional numerical search, in certain cases, a closed-form solution is possible. In Section IV, we will give examples of the optimal number of quantization bits when $\epsilon(\gamma)$

is specified for different modulation and receiver formats, with $\gamma := \mathcal{E}_b/N_0$ denoting the bit-energy-to-noise ratio.

B. Optimizing the Energy Allocation per Bit

In the previous subsection, we assumed that each bit is allocated identical energy. However, observing that each bit in (4) has a different weight suggests that there is room to optimize the energy per bit. This motivates us to look for an optimal energy allocation scheme when the total number of bits N is fixed. Let us suppose that bit i is allocated a fraction x_i of the total energy $\mathcal{E}$ for $i = 1, ..., N$. Then, following the derivation of (5), we have:

$$\begin{aligned} \mathsf{E}[|A - \hat{A}|] &\leq 2^{-N} + \sum_{i=1}^{N} \mathsf{E}[|b_i - \hat{b}_i|]2^{-i} \\ &= 2^{-N} + \sum_{i=1}^{N} \epsilon(\frac{\mathcal{E}x_i}{N_0})2^{-i}. \end{aligned} \tag{7}$$

In order to account for the mean-absolute reconstruction error with respect to $\mathbf{x} := [x_1, ..., x_N]^T$, we can formulate the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & f_0(\mathbf{x}) := 2^{-N} + \sum_{i=1}^{N} \epsilon(\frac{\mathcal{E}x_i}{N_0})2^{-i} \qquad (8) \\ \text{subject to} \quad & f_i(\mathbf{x}) := -x_i \leq 0,\ i = 1, ..., N, \\ & h(\mathbf{x}) := \sum_{i=1}^{N} x_i = 1. \end{aligned}$$

Letting $\mathbf{x}^* := [x_1^*, x_2^*, ..., x_N^*]^T$ denote the optimal solution, the well known Karush-Kuhn-Tucker (KKT) conditions [11, p. 243] dictate that there must exist $\{\lambda_i^*\}_{i=1}^N$ and ν^* such that:

$$x_i^* \geq 0,\ \lambda_i^* \geq 0,\ \lambda_i^* x_i^* = 0,\ i = 1, 2, ..., N; \tag{9}$$

$$\sum_{i=1}^{N} x_i^* = 1; \tag{10}$$

$$\nabla f_0(\mathbf{x}^*) + \sum_{i=1}^{N} \lambda_i^* \nabla f_i(\mathbf{x}^*) + \nu^* \nabla h(\mathbf{x}^*) = 0, \tag{11}$$

where ∇ denotes the gradient. It follows from (11) that the $\{x_i^*\}_{i=1}^N$ must satisfy:

$$2^{-i} \frac{\mathcal{E}}{N_0} \frac{d\epsilon(\gamma)}{d\gamma}\bigg|_{\gamma = \frac{\mathcal{E}}{N_0}x_i^*} - \lambda_i^* + \nu^* = 0, \quad i = 1, ..., N. \tag{12}$$

In order to gain further insight from (12), let us take a closer look at the optimal energy allocation in two special cases.

1) BPSK over AWGN Channel: The crossover probability ϵ expressed in terms of bit-energy-to-noise ratio γ is given in this case by [12, p. 255]:

$$\epsilon(\gamma) = Q(\sqrt{2\gamma}) := \int_{\sqrt{2\gamma}}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt.$$

The derivative of $\epsilon(\gamma)$ with respect to γ is then calculated as follows:

$$\frac{d\epsilon(\gamma)}{d\gamma} = -\frac{1}{\sqrt{2\pi}} \frac{e^{-\gamma}}{\sqrt{2\gamma}} := -\phi(\gamma). \tag{13}$$

Substituting (13) into (12), we can express the optimal energy allocation in closed form:

$$x_i^* = \phi^{-1}\left((\nu^* - \lambda_i^*)2^i \frac{N_0}{\mathcal{E}}\right) \frac{N_0}{\mathcal{E}}, \quad i = 1, ..., N. \tag{14}$$

Noticing that the domain of $\phi(\gamma)$ defined in (13) is $(0, +\infty)$, from the complementary slackness conditions in (9), we deduce that $\lambda_i^* = 0, \forall i$; and finally, we obtain:

$$x_i^* = \phi^{-1}\left(\nu^* 2^i \frac{N_0}{\mathcal{E}}\right) \frac{N_0}{\mathcal{E}}, \quad i = 1, ..., N, \tag{15}$$

where ν^* is a constant chosen such that $\sum_{i=1}^N x_i^* = 1$. Eq. (15) is intuitively appealing, because the monotonicity of $\phi(\gamma)$ ensures that each bit is allocated energy according to its significance: the smaller the i, the more significant bit i is and the more energy is allocated to bit i.

2) Binary Orthogonal Signaling with Envelope Detection: It is well known that binary orthogonal signals such as binary frequency-shift keying (FSK) or pulse-position modulation (PPM) can be demodulated using non-coherent envelope detection [12, pp. 307-310]. In this case, the crossover probability expressed in terms of the bit-energy-to-noise ratio is given by:

$$\epsilon(\gamma) = \frac{1}{2} e^{-\frac{\gamma}{2}}.$$

The derivative is then:

$$\frac{d\epsilon(\gamma)}{d\gamma} = -\frac{1}{4} e^{-\frac{\gamma}{2}} := -\varphi(\gamma). \tag{16}$$

Substituting (16) into (12), we obtain:

$$x_i^* = \varphi^{-1}\left((\nu^* - \lambda_i^*)2^i \frac{N_0}{\mathcal{E}}\right) \frac{N_0}{\mathcal{E}}. \tag{17}$$

Noticing that the function $\varphi(\gamma)$ has domain $[0, +\infty)$ and range $(0, \varphi(0) = 1/4]$, and supposing that $\lambda_i^* = 0, \forall i$, we must have $\nu^* 2^N N_0/\mathcal{E} \leq 1/4$. Furthermore, the condition $\sum_{i=1}^N x_i^* = 1$ is not guaranteed to be satisfied when ν^* is bounded. Based on (9), we can simplify (17) as follows:

$$\begin{aligned} x_i^* &= \varphi^{-1}\left(\min\left\{\frac{1}{4},\ \nu^* 2^i \frac{N_0}{\mathcal{E}}\right\}\right) \frac{N_0}{\mathcal{E}} \\ &= 2\frac{N_0}{\mathcal{E}} \ln\left(\frac{1}{4\left(\min\left\{\frac{1}{4},\ \nu^* 2^i \frac{N_0}{\mathcal{E}}\right\}\right)}\right). \end{aligned} \tag{18}$$

Eq. (18) implies that it is possible to have $x_i^* = 0$ for some large i's. In fact, when $\epsilon(\gamma) = (1/2)e^{-\gamma/2}$, the problem in (8) can be readily shown to be convex which not only implies that the optimal solution is guaranteed to exist and is unique, but also it can be found using a numerically efficient search.

The optimal energy allocation for a special case will also be re-visited in Section IV, where we will confirm that a certain number of less significant bits should not be allocated any energy.

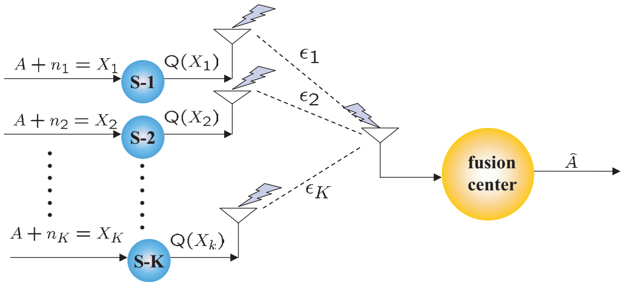

Fig. 2. Multi-sensor cooperation in estimating a scalar parameter with quantized observations.

III. Multi-Sensor Cooperation in Estimating a Parameter

Let us now consider the multi-sensor setup depicted in Fig. 2, where each sensor k has available a local noisy observation $X_k = A + n_k$, and n_k is zero mean with variance σ_k^2 and independent of n_l for $l \neq k$. After normalization, we have $X_k \in [0,1]$. Sensor k quantizes its local observation X_k to the N most significant bits, i.e., with $X_k = \sum_{i=1}^{\infty} b_i^{(k)} 2^{-i}$, we have $(X_k)_Q = \sum_{i=1}^{N} b_i^{(k)} 2^{-i}$. Bits $\{b_i^{(k)}\}_{i=1}^{N}$ are then transmitted through the wireless channel, which is again modelled as a BSC with crossover probability ϵ_k. The fusion center reconstructs X_k with the received bits $\{\hat{b}_i^{(k)}\}_{i=1}^{n}$ to obtain:

$$\hat{X}_k = \sum_{i=1}^{N} \hat{b}_i^{(k)} 2^{-i}.$$

When we have available un-quantized real valued observations: $X_k = A + n_k, k = 1, 2, ..., K$, the best linear unbiased estimator (BLUE) of A is known to be [13]:

$$\hat{A}_{BLUE} = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \sum_{k=1}^{K} \frac{X_k}{\sigma_k^2}.$$

This motivates us to form the following estimator for the parameter A when the noise variances are known at the fusion center, where we have only available $\hat{X}_k, k = 1, 2, ..., K$:

$$\hat{A} = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \sum_{k=1}^{K} \frac{\hat{X}_k}{\sigma_k^2}. \tag{19}$$

The problem we are interested in can be formulated as follows: *For a fixed number of quantization bits (N_k) per sensor, what is the optimal scheme for allocating the total energy E_T prescribed to all sensors so that the mean-square estimation error $\mathsf{E}|\hat{A} - A|^2$ is minimized? Furthermore, what is the optimal number of quantization bits per sensor so that this energy allocation scheme achieves the minimum estimation error?*

For clarity in exposition, we only consider here a simple situation where each sensor transmits the same fixed number of bits N (i.e., $N_k = N, \forall k$), and the energy allocated per sensor is equally distributed among bits. Now, let us take a look at the estimation error:

$$\begin{aligned}\hat{A} - A &= \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \sum_{k=1}^{K} \frac{\hat{X}_k - A}{\sigma_k^2} \\ &= \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \sum_{k=1}^{K} \frac{\hat{X}_k - X_k + n_k}{\sigma_k^2}.\end{aligned}$$

Upon defining the reconstruction error $\tilde{X}_k := \hat{X}_k - X_k$, we have:

$$\mathsf{E}|\hat{A} - A|^2 = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-2} \mathsf{E}\left|\sum_{k=1}^{K} \frac{\tilde{X}_k + n_k}{\sigma_k^2}\right|^2 = \frac{\mathsf{E}\left|\sum_{k=1}^{K} \frac{\tilde{X}_k}{\sigma_k^2}\right|^2 + \mathsf{E}\left[\left(\sum_{k=1}^{K} \frac{\tilde{X}_k}{\sigma_k^2}\right)\left(\sum_{k=1}^{K} \frac{n_k}{\sigma_k^2}\right)\right] + \sum_{k=1}^{K} \frac{1}{\sigma_k^2}}{\left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^2}.$$

Because the observation noise n_k can be safely assumed uncorrelated with the reconstruction error $\tilde{X}_k$, the middle term of the numerator disappears. Hence, minimizing $\mathsf{E}|A - \hat{A}|^2$ reduces to minimizing $\mathsf{E}\left|\sum_{k=1}^{K} \frac{\tilde{X}_k}{\sigma_k^2}\right|^2$. Because for any bounded random variable $Z \in [-U, U]$ with pdf $p(z)$, we have $\mathsf{E}|Z|^2 = \int_{-U}^{U} |z|^2 p(z) dz \leq \int_{-U}^{U} U|z| p(z) dz = U\mathsf{E}|Z|$, noticing that $\sum_{k=1}^{K} \frac{\tilde{X}_k}{\sigma_k^2}$ is bounded, we can instead minimize $\mathsf{E}\left|\sum_{k=1}^{K} \frac{\tilde{X}_k}{\sigma_k^2}\right|$, which we upper bound as:

$$\mathsf{E}\left|\sum_{k=1}^{K} \frac{\tilde{X}_k}{\sigma_k^2}\right| \leq \sum_{k=1}^{K} \frac{\mathsf{E}|\tilde{X}_k|}{\sigma_k^2} \leq \sum_{k=1}^{K} \frac{2^{-N} + (1 - 2^{-N})\epsilon_k}{\sigma_k^2},$$

where ϵ_k is the crossover probability of the BSC between sensor k and the fusion center. With x_k denoting the fraction of the total energy E_T allocated to sensor k, we can express ϵ_k as $\epsilon_k = \epsilon_k(\frac{E_T x_k}{N N_0})$, where N_0 is the noise level at the receiver of the fusion center that is assumed common to all channels. The optimal energy allocation scheme will be the solution of the following optimization problem ($\mathbf{x} := [x_1, ..., x_K]^T$):

$$\begin{aligned} &\text{minimize} && f_0(\mathbf{x}) := \sum_{k=1}^{K} \frac{\frac{1}{2^N} + (1 - \frac{1}{2^N})\epsilon_k\left(\frac{E_T x_k}{N N_0}\right)}{\sigma_k^2} \\ &\text{subject to} && f_k(\mathbf{x}) := -x_k \leq 0, \;\; k = 1, ..., K, \\ & && h(\mathbf{x}) := \sum_{k=1}^{K} x_k = 1. \end{aligned} \tag{20}$$

As in Section II, we can write down the KKT conditions for the optimal solution $\mathbf{x}^* := [x_1^*, ..., x_K^*]^T$ as follows:

$$x_k^* \geq 0, \;\; \lambda_k^* \geq 0, \;\; \lambda_k^* x_k^* = 0, \;\; k = 1, 2, ..., K; \tag{21}$$

$$\sum_{k=1}^{K} x_k^* = 1; \tag{22}$$

$$\nabla f_0(\mathbf{x}^*) + \sum_{k=1}^{K} \lambda_k^* \nabla f_k(\mathbf{x}^*) + \nu^* \nabla h(\mathbf{x}^*) = 0. \tag{23}$$

From (23), we have:

$$\frac{1}{\sigma_k^2}\frac{E_T}{NN_0}\frac{d\epsilon_k(\gamma)}{d\gamma}\bigg|_{\gamma=\frac{E_T}{NN_0}x_k} - \lambda_k^* + \nu^* = 0, \qquad k = 1, ..., K. \tag{24}$$

When $\epsilon_k(\gamma), \forall k$ is convex[1] in γ, the problem in (20) turns out to be convex which implies that the global optimum exists and can be easily found numerically. Let us now illustrate the approach of this section using a simple example. Letting κ denote the path loss exponent [14] of the wireless channel, d_k the distance between sensor k and the fusion center, and supposing BPSK modulation, we can express the crossover probability in the presence of AWGN as: $\epsilon_k(\gamma) = Q(\sqrt{2\gamma\mathcal{C}/d_k^\kappa})$ with $\mathcal{C}$ being a constant. Under these operating conditions, (24) and (21) yield:

$$\begin{aligned} x_k^* &= \phi_k^{-1}\left(\nu^*\sigma_k^2\frac{NN_0}{E_T}\right)\frac{NN_0}{E_T}, \qquad (25)\\ \phi_k(\gamma) &:= \frac{1}{\sqrt{2\pi}}\frac{e^{-\gamma\frac{\mathcal{C}}{d_k^\kappa}}}{\sqrt{2\gamma}}\sqrt{\frac{\mathcal{C}}{d_k^\kappa}}, \end{aligned}$$

where ν^* is chosen such that $\sum_{k=1}^K x_k^* = 1$. In Section IV, we will examine a specific system and find the corresponding optimal energy allocation to gain more insights into these closed-form expressions.

The optimal number of quantization bits N_{opt} can be easily found using one-dimensional numerical search to solve the optimization problem:

$$N_{opt} = \arg\min_N f_0(\mathbf{x}^*), \tag{26}$$

where $f_0(\mathbf{x}^*)$ is the optimal value of the objective function in (20) when the number of quantization bits for each sensor is N. In Section IV, we will show an example of the functional relationship between $f_0(\mathbf{x}^*)$ and N, from which N_{opt} can be readily determined.

IV. Numerical Examples

In this section, we provide numerical examples to corroborate the analytical results we derived in the previous sections.

A. Optimal Number of Quantization Levels

As discussed in Section II-A, there exists an optimal value of N which minimizes the mean-absolute reconstruction error upper bound. Here, we consider the channel to be AWGN and use BPSK modulation. The BSC crossover probability as a function of the bit-energy-to-noise ratio is of the form $\epsilon(\gamma) = Q(\sqrt{2\gamma})$. Fig. 3 depicts the bound $f(N)$ in (5) with $\mathcal{E}/N_0 = 20$, whose numerical minimization yields $N_{opt} = 7$. In Fig. 3, we also plot $f(N)$ when $\epsilon(\gamma) = (1/2)e^{-\gamma/2}$, which is the BER when binary orthogonal modulation is used along with envelope detection. The optimum number in this case is $N_{opt} = 5$.

[1] In most cases, convexity is guaranteed when $\epsilon_k(\gamma)$ is expressible in terms of $Q(\sqrt{2\gamma})$ or $(1/2)e^{-\gamma/2}$.

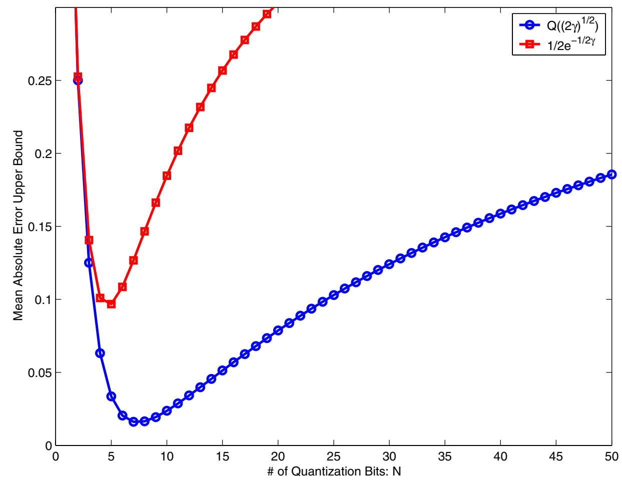

Fig. 3. The bound in (5) whose minimum yields the optimum number of quantization bits.

B. Optimal Bit Energy Allocation

In Section II-B, we derived an optimal energy allocation scheme per bit to minimize the reconstruction error. Considering envelope detection of binary orthogonal signals as in Section II-B.2, with $\mathcal{E}/N_0 = 20$ and $N = 10$, we find the optimal energy allocation by solving the convex optimization of (8) using the interior-point method [11, chap. 11], and depict it in Fig. 4.

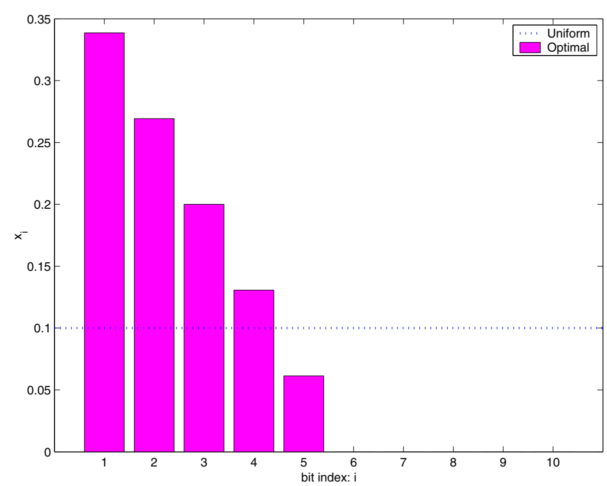

Fig. 4. Optimal energy allocation over a fixed number of quantization bits ($N = 10$).

For the same $\epsilon(\gamma) = (1/2)e^{-\gamma/2}$, Fig. 5 compares the reconstruction error between the optimal energy allocation scheme and the equal energy distribution scheme with different number of quantization bits N. We observe that the reconstruction error decreases to a floor as N increases with optimal energy allocation, which is different from the equal energy allocation scheme. The explanation for this behavior is that as N increases, equal energy allocation increases the crossover probability for all transmitted bits; on the other hand, optimal energy allocation will not experience this problem. As already noticed in Fig. 4, when N is large enough, the optimal scheme just assigns no (or very little) energy to less significant bits.

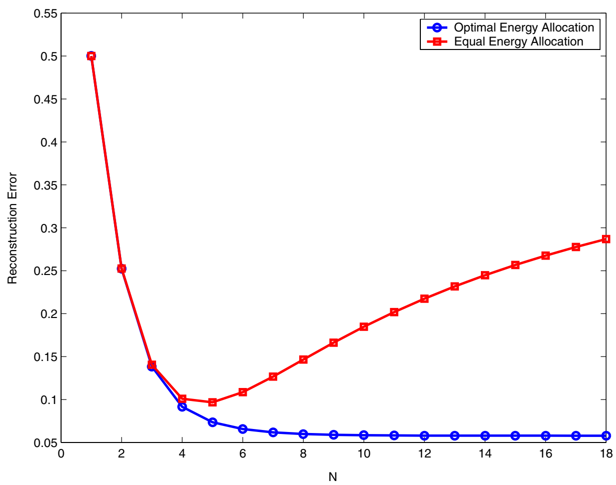

Fig. 5. Comparison between equal energy allocation and optimal energy allocation.

C. Optimal Energy Allocation among Sensors

Suppose that $K = 10$ sensors are deployed as shown in Fig. 6 with the local observation noise variances denoted by $\sigma_1^2, \sigma_2^2, ..., \sigma_{10}^2$; the path loss exponent of the wireless channel is $\kappa = 2$ (free space); and, accordingly, the crossover probability is given by $\epsilon_k(\gamma) = Q\left(\sqrt{2\gamma \mathcal{C}/d_k^2}\right)$ and $\mathcal{C}$ is set to be 1 here. The total energy budget is set to be $E_T/N_0 = 200$.

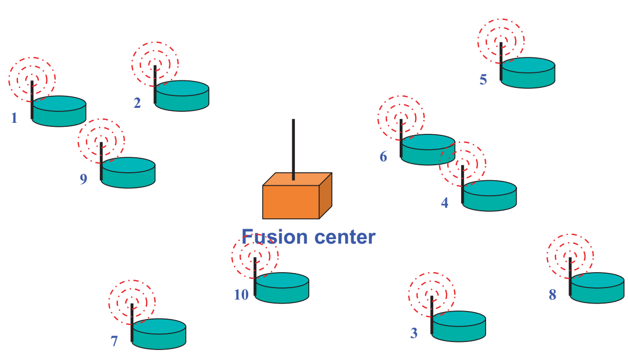

Fig. 6. Sensor deployment topology.

With different sets of values for $\{d_k\}_{k=1}^{10}$ and $\{\sigma_k^2\}_{k=1}^{10}$, using the above system parameters, the numerical solution of the convex problem in (20) is depicted in Fig. 7. Subsequently, Fig. 8 compares the normalized value of the objective function in (20) between equal energy allocation and the optimal energy allocation scheme for a variable number of bits: N while choosing a specific set of values for $\{d_k\}_{k=1}^{10}$ and $\{\sigma_k^2\}_{k=1}^{10}$.

It is interesting to see that in Fig. 7, under CASE I, the fraction of energy allocated to a sensor will not always increase

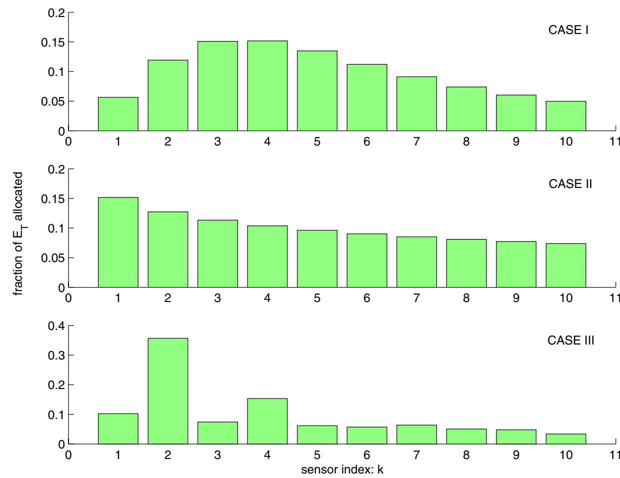

Fig. 7. Optimal energy allocation scheme when using the corresponding optimal number of quantization bits: N_{opt}. CASE I. $d_k = k$, $\sigma_k^2 = 0.01, \forall k$. CASE II. $d_k = 1$, $\sigma_k^2 = 0.01 \times k, \forall k$. CASE III. $\sigma_k^2 = 0.01 \times k, \forall k$ and, $\{d_1, d_2, ..., d_{10}\} = \{1, 5, 1, 5, 1, 1, 5, 5, 1, 5\}$.

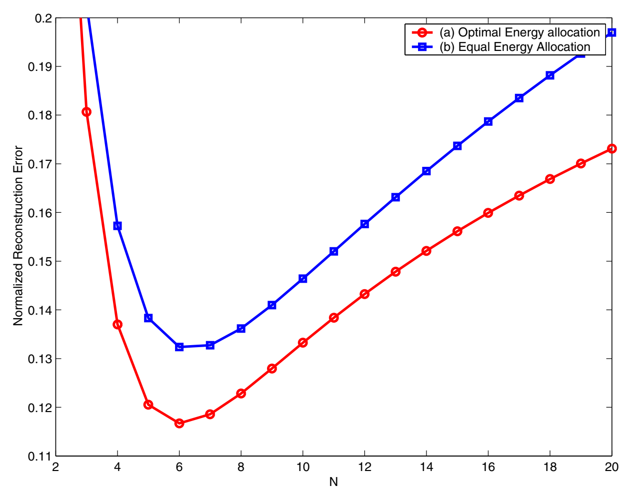

Fig. 8. With $\sigma_k^2 = 0.01 \times k, \forall k$ and, $\{d_1, d_2, ..., d_{10}\} = \{1, 5, 1, 5, 1, 1, 5, 5, 1, 5\}$: (a) Normalized value of the objective function in (20) with optimal energy allocation among sensors. (b) Normalized value of the objective function in (20) with equal energy allocation among sensors.

when the distance between the sensor and the fusion center is increasing. In fact, this is attributed to the property of $\phi_k(\gamma)$, which is defined in (25).

V. Conclusions

Motivated by stringent energy requirements that are prevalent in wireless sensor networks, we have pursued optimal quantization of physical quantities at sensor nodes to effect optimal reconstruction at the fusion center. (De-)modulation and propagation were captured through the use of binary symmetric channel modelling. In particular, we have found the number of quantization bits for minimizing the mean-absolute reconstruction error bound in a point-to-point link when each bit is allocated the same energy. We also derived an optimal scheme for energy allocation across quantization bits. When multiple sensors collaborate to estimate a parameter in noise, we also obtained the optimal energy allocation scheme to distribute the limited total energy among different sensors when each sensor assigns the same energy to all its quantization bits. It turned out that this allocation scheme depends on the prescribed number of quantization bits N and thus, the optimal N can also be found with the help of our convex optimization formulation.

Our future work will analyze the effect of error control coding on the optimal quantizer design under energy constraints.

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