

Tools for Performance Analysis and Design of Space–Time Block Codes

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Abstract—Space–time block codes (STBCs) have attracted recent interest due to their ability to take advantage of both space and time diversity to reliably transmit data over a wireless fading channel. In many cases, their design is based on asymptotically tight performance criteria, such as the worst-case pairwise error probability (PEP) or the union bound. However, these quantities fail to give an accurate performance picture, especially at low signal-to-noise ratio, because the classical union bound is known to be loose in this case. This paper develops tighter performance criteria for STBCs which yield considerably better bounds. First, the union bound is developed as the average of the exact PEPs. By noting that some of the terms in the bound are redundant, a second bound is obtained by expurgation. Since this still yields a loose bound, a tighter bound, denoted as the progressive union bound (PUB), is obtained. Because the PUB cannot be computed in closed form, in its most general case, and to avoid computing a high-dimensional numerical integration, its saddlepoint approximation is developed. In addition to the significant improvement of the PUB analysis over other bounding methods, it is also shown that codes designed to optimize the PUB can perform better than those obtained by the looser criteria.

Index Terms—Block codes, diversity methods, fading channels, multiple-input multiple-output (MIMO) systems, performance analysis.

I. INTRODUCTION

SPACE–TIME block coding has attracted considerable attention recently as a technique that employs diversity to mitigate the adverse effects of fading in wireless channels. This has been proven to yield a dramatic increase in achievable data rates (also known as multiplexing gain) and communication reliability (i.e., diversity gain) in multiple-input/multiple-output (MIMO) systems [1], [2].

Inspired by the results in [1], several space–time block code (STBC) schemes, such as orthogonal [3] and unitary group [4], [5] codes, have been proposed, by enforcing a certain structure

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on the codewords to take full advantage of diversity. On the other hand, unstructured designs, such as optimum minimum metric (OMM) [6] and union bound [7] codes found by computer search can offer large performance gains in comparison with structured approaches. More recently, a hybrid scheme employing limited computer searches combined with a hierarchical codeset construction has been shown to enable construction of good high-rate codes in a computationally feasible manner [8], yielding codes that offer improved performance over previously proposed codes.

While STBC design has evolved considerably over the recent years, there is still a lack of analytical results for accurate performance assessment of codes. Due to the nonexistence of simple expressions for the block-error rate of codesets in general, most of the previous work relied on the pairwise error probability (PEP) bound as simple performance criterion [1], [9], [10].

The first proposed method for bounding the worst-case PEP of STBCs is the classical Chernoff bound approach [1], [11]. More recently, some works have proposed tighter upper bounds on the PEP applied to space–time trellis (STT) coding schemes. The work in [12] presents an upper bound based on Craig’s form for the Gaussian tail function. Although tighter than the bounds in [1] and [11], it requires a numerical integration to be computed. This is also the case in [13], where a matched-filter bound is developed for a system in frequency-selective fading employing binary phase-shift keying.

The Chernoff bound analysis yields the well-known rank and determinant criteria for “optimal” code construction. As the code size increases, however, the worst-case error probability is not sufficient to characterize the full picture of performance [7]. Therefore, approaches based on the union bound have recently been applied to the analysis of nonlinear [7] STBCs. On the other hand, the union bound for encoders employing a linear structure has been studied in [14], where it was proved that orthogonal codes yield optimal performance among unitary codes by achieving the lowest bound. In [15], a union-bound criterion for STBC over channels with intersymbol interference is also considered. We note that the union bound has also been investigated in the context of STT codes [10], [16].

Although asymptotically tight, the union bound is quite loose at low signal-to-noise ratios (SNRs), due to the large number of overlapping of decision regions in the PEP computation. We, therefore, present a PEP-expurgation method resulting in a tighter bound, the indecomposable union bound (IUB). The achievable expurgation, however, can differ considerably among different codesets.

In this paper, we develop, through a unified approach, several upper bounds on the performance of STBCs. Rather than

considering only the worst-case PEP, our bounds take the entire distance spectrum of the codes into consideration, resulting in improved performance assessment. Under this framework, the union bound and IUB are derived. Moreover, we propose a new, inherently tighter performance bound, which is developed from the progressive union bound (PUB) concept. First presented in [18], the PUB was used to analyze the performance of nonspread transmitter-diversity schemes employing intentional frequency offset, and its computation was performed via a numerical integration. Our work applies the PUB to the analysis of spread and nonspread STBC-based systems. We adopt a different approach to the PUB computation, by deriving a saddlepoint (SP) approximation of this quantity. This method has the advantage of being computationally more attractive than the numerical integration, while also allowing for a semianalytical expression to be derived for the PUB. Furthermore, it can be applied to code design by searching for codes that minimize this PUB approximation. Finally, the PUB allows a tradeoff between accuracy and numerical complexity by varying the parameters of the PUB computation.

This paper is organized as follows. In Section II, the STBC system model is presented. Section III develops the union bound performance criterion. The IUB is presented in Section IV and some of its properties are discussed. Section V develops the PUB and its SP approximation, and Section VI presents a brief review of a simple code construction technique. We next apply the analysis criteria discussed herein to code design by providing tables of new found codes, along with performance comparisons in Section VII. Concluding remarks are presented in Section VIII.

II. SIGNAL MODEL

We consider a general single-user system model encompassing both spread and nonspread systems. The terminology *spread* and *nonspread* refers to the use or absence, respectively, of possibly distinct spreading codes at each of the transmit antennae. Thus, this very general model is easily extensible to multiuser spread-spectrum systems, while also having utility for single-user systems without signal spreading. We observe that for systems where the transmission bandwidth exceeds the coherence bandwidth of the channel, as often experienced by spread-spectrum systems, the appropriate channel model is that of a multipath channel. However, in this paper, for both spread and nonspread systems, we shall focus on channels with a single (dominant) flat-fading path component at the receiver. This assumption is motivated by a desire to keep the notation simple (all of our methods are easily extensible to multipath channels), and by the observation that codes optimized for flat-fading channels also provide good performance in multipath channels [19], [20]. Spreading allows for additional signal separation and therefore improved performance.

The transmitter, equipped with L_t antennae, maps a vector $[I(1), I(2), \dots, I(k_c)]$ of k_c information bits to one of $K_c = 2^{k_c}$ space-time codewords, $[d_i(t)]$, $1 \leq i \leq L_t$ and $1 \leq t \leq N_c$. The block length, in terms of bit duration, is N_c , resulting in a transmission matrix \mathcal{D} of size $N_c \times L_t$ and code rate of

$R_c = k_c/N_c$. Each element $d_i(t)$ of \mathcal{D} is spread by a corresponding spreading code $\mathbf{s}_i(t)$ and transmitted via the corresponding transmit antenna TX_i . The receiver is equipped with L_r antennae. Note that each row of the codeword matrix is transmitted simultaneously. In this paper, each $d_i(t)$ is constrained to phase-shift keying (PSK) constellations, but, in general, can be taken from any point on the complex plane. The spreading waveform $\mathbf{s}_i(t)$ for antenna i is sampled at the chip rate $1/T_c$ to form a column vector of length L_u , denoted by $\mathbf{s}_i(n)$. Different spreading codes can be used at each antenna.

Assuming synchronous transmission, the received signal (output of a matched filter at the receiver) at time n and antenna i can be written as

$$\mathbf{y}_i(n) = \sqrt{\sigma_t} R(n) D(n) \mathbf{h}_i(n) + \mathbf{m}_i(n) \in \mathbb{C}^{L_t \times 1} \quad (1)$$

where σ_t is SNR normalized by L_t (i.e., $\sigma_t = \text{SNR}/L_t$), to keep the total transmit power constant and

$$S(n) = [\mathbf{s}_1(n), \dots, \mathbf{s}_{L_t}(n)] \in \mathbb{R}^{L_u \times L_t} \quad (2)$$

$$R(n) = S(n)^T S(n) \in \mathbb{R}^{L_t \times L_t} \quad (3)$$

$$D(n) = \text{diag}(d_1(n), \dots, d_{L_t}(n)) \in \mathbb{C}^{L_t \times L_t} \quad (4)$$

$$\mathbf{h}_i(n) = [h_{i1}(n), \dots, h_{iL_t}(n)]^T \in \mathbb{C}^{L_t \times 1} \quad (5)$$

where $R(n)$ is the spreading code correlation matrix at time n (for nonspread systems, $R(n)$ assumes the form of an all-ones matrix), $\mathbf{h}_i(n) \sim \mathcal{CN}(0, I_{L_t})^1$ is the channel coefficient vector at time n for the i th receive antenna, and $\mathbf{m}_i(n) \sim \mathcal{CN}(0, R(n))$ is the received complex Gaussian noise vector at time n for the i th receive antenna. Note that $D(n)$ is obtained by diagonalization of the n th row of codeword \mathcal{D} .

We assume a quasi-static fading channel, $\mathbf{h}_i(1) = \dots = \mathbf{h}_i(N_c)$. Concatenating $\mathbf{y}_i(n)$ and $\mathbf{m}_i(n)$, $n = 1, \dots, N_c$ into larger vectors \mathbf{y}_i and \mathbf{m}_i , respectively (such that $\mathbf{y}_i \triangleq [\mathbf{y}_i^T(1), \dots, \mathbf{y}_i^T(N_c)]^T$, and similarly for \mathbf{m}_i), we get

$$\mathbf{y}_i = \sqrt{\sigma_t} \begin{bmatrix} R(1) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & R(N_c) \end{bmatrix} \begin{bmatrix} D(1) \\ \vdots \\ D(N_c) \end{bmatrix} \mathbf{h}_i(1) + \mathbf{m}_i \quad (6)$$

$$\triangleq \sqrt{\sigma_t} \mathbf{R} \mathbf{D} \mathbf{h}_i + \mathbf{m}_i \in \mathbb{C}^{L_t N_c \times 1} \quad (7)$$

where $\mathbf{R} \in \mathbb{R}^{L_t N_c \times L_t N_c}$ is the spreading code correlation matrix, $\mathbf{D} \in \mathbb{C}^{L_t N_c \times L_t}$ is the transmitted codeword matrix, and $\mathbf{h}_i \triangleq \mathbf{h}_i(1) \in \mathbb{C}^{L_t \times 1}$ is the channel coefficient vector for receive antenna i .

We now vertically concatenate the vectors \mathbf{y}_i , $i = 1, \dots, L_r$ to form the vector $\bar{\mathbf{y}} \in \mathbb{C}^{L_t N_c L_r \times 1}$. Similarly, we also concatenate vectors \mathbf{h}_i and \mathbf{m}_i , obtaining $\bar{\mathbf{h}} \in \mathbb{C}^{L_t L_r \times 1}$ and $\bar{\mathbf{m}} \in \mathbb{C}^{L_t N_c L_r \times 1}$, respectively. The resulting signal model becomes

$$\bar{\mathbf{y}} = \sqrt{\sigma_t} \begin{bmatrix} \mathbf{R} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{D} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_{L_r} \end{bmatrix} + \begin{bmatrix} \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_{L_r} \end{bmatrix} \quad (8)$$

$$\triangleq \sqrt{\sigma_t} \bar{\mathbf{R}} \bar{\mathbf{D}} \bar{\mathbf{h}} + \bar{\mathbf{m}} \in \mathbb{C}^{L_t N_c L_r \times 1} \quad (9)$$

¹We use $\mathcal{CN}(\mathbf{m}, K)$ to denote a circularly symmetric complex Gaussian random vector with mean \mathbf{m} and variance matrix K .

where $\bar{\mathbf{R}} \in \mathbb{C}^{L_t N_c L_r \times L_t N_c L_r}$ and $\bar{D} \in \mathbb{C}^{L_t N_c L_r \times L_t L_r}$.

In the following sections, we develop expressions for performance analysis of STBCs considering the general signal model developed in this section. First, we introduce the classical Chernoff bound on the PEP and the union bound. Subsequently, we will show that the union bound can be tightened using the notion of indecomposable error patterns, yielding a third criterion, the IUB. All three bounds are asymptotically tight as the SNR increases, but are loose at low SNR. With this motivation in mind, we propose a fourth performance criterion, the PUB, which better predicts code performance at low SNR.

III. THE CHERNOFF AND UNION BOUNDS

We define the (normalized) difference between any pair of transmitted codewords as an *error pattern*. Thus, the set of error patterns that affect the k th codeword is

$$E_k \triangleq \left\{ \mathbf{e}_{kj} | \mathbf{e}_{kj} = \left(\frac{\bar{D}_k - \bar{D}_j}{2} \right) \quad \forall j \neq k \right\}. \quad (10)$$

For a synchronous system, at high SNR, the average probability of decoding D_β when D_α is transmitted is upper bounded by the asymptotically tight Chernoff bound [1], [9]

$$E_{\mathbf{h}}[P(D_\alpha \rightarrow D_\beta | \mathbf{h})] \leq \left(\frac{\sigma_t}{4} \right)^{-L_t L_r} \frac{1}{|\Phi(\alpha, \beta, \beta)|} \quad (11)$$

where $|\Phi(\alpha, \beta, \beta)| \triangleq \det \Phi(\alpha, \beta, \beta)$. The generalized correlated codeword difference matrix (assumed in (11) to be full rank) $\Phi(\alpha, \beta, \gamma)$ is defined as

$$\Phi(\alpha, \beta, \gamma) \triangleq 4(\mathbf{e}_{\alpha\beta})^H \bar{\mathbf{R}} \mathbf{e}_{\alpha\gamma} \in \mathbb{C}^{L_t L_r \times L_t L_r}. \quad (12)$$

If we assume fixed spreading codes are used within one block $R(1) = \dots = R(N_c) \triangleq R$, and define²

$$\Psi(\alpha, \beta, \gamma) = (\mathcal{D}_\alpha - \mathcal{D}_\beta)^H (\mathcal{D}_\alpha - \mathcal{D}_\gamma) \odot R \in \mathbb{C}^{L_t \times L_t} \quad (13)$$

then $\Phi(\alpha, \beta, \gamma)$, can be rewritten as

$$\Phi(\alpha, \beta, \gamma) = \text{diag}[\Psi(\alpha, \beta, \gamma), \dots, \Psi(\alpha, \beta, \gamma)] \in \mathbb{C}^{L_t L_r \times L_t L_r}. \quad (14)$$

In contrast to the simple bound given in (11), the average PEP can be calculated exactly in closed form by $E_{\mathbf{h}}[P(D_\alpha \rightarrow D_\beta | \mathbf{h})] = \theta(\alpha, \beta)$, where $\theta(\alpha, \beta)$ is a function of the eigenvalues of $(\sigma_t/4)\Phi(\alpha, \beta, \beta)$. For example, in the case of M distinct eigenvalues (see, e.g., [19] and [20])

$$\theta(\alpha, \beta) = \frac{1}{2} \sum_{i=1}^M \left(\prod_{j=1, j \neq i}^M \frac{\lambda_i}{\lambda_i - \lambda_j} \right) \left(1 - \frac{1}{\sqrt{1 + 2/\lambda_i}} \right). \quad (15)$$

We denote the union bound performance index, UB, as

$$\text{UB} \triangleq \sum_{k=1}^{K_c} \sum_{j=1}^{K_c} \theta(k, j) = \sum_{k=1}^{K_c} \sum_{\mathbf{e}_{kj} \in E_k} \theta(k, j). \quad (16)$$

² \odot denotes Schur product, i.e., elementwise multiplication.

At high SNR, the Chernoff bound in (11) can be used. Hence, the symbol-error rate (SER) can be bounded as

$$\text{SER} \leq \frac{1}{K_c} \text{UB} \leq \frac{1}{K_c} \sum_{i=1}^{K_c} \sum_{j=1, j \neq i}^{K_c} \left(\frac{\sigma_t}{4} \right)^{-L_t} \frac{1}{|\Phi(i, j, j)|}. \quad (17)$$

Assuming all codeword pairs achieve full diversity, the following quantity is a scaled version of the bound in (17), and independent of the SNR:

$$\text{CB} \triangleq \sum_{i=1}^{K_c} \sum_{j=i+1}^{K_c} \frac{1}{|\Phi(i, j, j)|}. \quad (18)$$

Since the union bound is known to be loose at low SNR, a more accurate performance index is desired. We next develop a method for obtaining of a tighter union bound, considering the fact that some of the PEP terms in (16) can be redundant, and thus, can be discarded.

IV. INDECOMPOSABLE UNION BOUND

In the first part of this section, we introduce the main definitions and extend the notion of error pattern decomposability presented in [21] for additive white Gaussian noise channels to STBCs. In the second part, these definitions are further extended to flat-fading channels and a new bound based on these patterns is developed, which is inherently tighter than (16). Finally, some properties of indecomposable patterns under flat-fading channels are presented.

A. Basic Definitions

Let $\mathcal{C} = \{\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_{K_c}\}$ be the set of codewords. Each row of the elements of this set is diagonalized, to form the set of transmitted codewords $\mathcal{C}' = \{D_1, D_2, \dots, D_{K_c}\}$.

We now define the weighted inner product between two error patterns \mathbf{e}_{km} and \mathbf{e}_{kn} of the k th codeword as

$$S(k, m, n) \triangleq \bar{\mathbf{h}}^H \Phi(k, m, n) \bar{\mathbf{h}}. \quad (19)$$

The squared norm of an error pattern \mathbf{e}_{kl} is thus given by

$$\|S(k, l, l)\|^2 = \bar{\mathbf{h}}^H \Phi(k, l, l) \bar{\mathbf{h}}. \quad (20)$$

Generalizing the definitions in [21] and [22], we have the following definition.

Definition 1: For a given $\bar{\mathbf{h}}$, an error pattern $\mathbf{e}_{kl} \in E_k$ is decomposable into patterns $\mathbf{e}_{km} \in E_k$ and $\mathbf{e}_{kn} \in E_k$ (denoted by $\mathbf{e}_{kl} \doteq \mathbf{e}_{km} + \mathbf{e}_{kn}$) if:

- 1) $\mathbf{e}_{kl} = \mathbf{e}_{km} + \mathbf{e}_{kn}$;
- 2) $[\mathbf{e}_{kl}]_{ij} = 0 \Leftrightarrow [\mathbf{e}_{km}]_{ij} = [\mathbf{e}_{kn}]_{ij} = 0$;
- 3) $\Re\{S(k, m, n)\} \geq 0$;

where $\Re\{\cdot\}$ denotes the real part. We also define

$$w(\mathbf{e}) \triangleq \sum_i \sum_j I([\mathbf{e}]_{ij} \neq 0) \quad (21)$$

where $I(\cdot)$ is the indicator function, taking the value 1 when its argument is true, and 0 otherwise. Note that if $\mathbf{e}_{kl} \doteq \mathbf{e}_{km} + \mathbf{e}_{kn}$ holds, an immediate consequence of condition 2) is that $w(\mathbf{e}_{km}) < w(\mathbf{e}_{kl})$ and $w(\mathbf{e}_{kn}) < w(\mathbf{e}_{kl})$.

We denote the set of *indecomposable* patterns in E_k by $F_k(\bar{\mathbf{h}})$. Thus

$$F_k(\bar{\mathbf{h}}) \triangleq \{ \text{indecomposable patterns under channel } \bar{\mathbf{h}} \} \subseteq E_k. \quad (22)$$

The definition of error patterns presented here differs from that given in [20] and [22] (in the context of multiuser detection), in the sense that all symbols in a codeword are taken into consideration when defining allowable error patterns, instead of only the symbol of the user of interest. With the above definitions, we can develop an expression for the union bound on PEP of a codeword error, for a given channel realization. Following a procedure similar to that of [20], the union bound of the PEP for codeword k conditioned on the channel is

$$P_k | \bar{\mathbf{h}} \leq \sum_{\mathbf{e}_{kl} \in E_k} Q(\sqrt{2\sigma_t} \|S(k, l, l)\|). \quad (23)$$

Furthermore, this bound can be tightened by expurgating the decomposable error patterns, resulting in

$$P_k | \bar{\mathbf{h}} \leq \sum_{\mathbf{e}_{kl} \in F_k(\bar{\mathbf{h}})} Q(\sqrt{2\sigma_t} \|S(k, l, l)\|). \quad (24)$$

Note that in this case, we sum over the smaller set of indecomposable patterns only.

We now average (24) over the channel statistics, resulting in

$$P_k \leq E_{\bar{\mathbf{h}}} \left\{ \sum_{\mathbf{e}_{kl} \in F_k(\bar{\mathbf{h}})} Q(\sqrt{2\sigma_t} \|S(k, l, l)\|) \right\}. \quad (25)$$

As observed in [22], the averaging in the right-hand side of (25) is intractable, due to the fact that each set $F_k(\bar{\mathbf{h}})$ depends on the particular channel realization $\bar{\mathbf{h}}$. Thus, our next step is to develop a channel-independent criterion for decomposability, which allows us to interchange the expectation and summation and obtain a closed-form upper-bound expression.

B. Decomposability in Quasi-Static Fading

In order to obtain a channel-independent criterion for decomposability, we place a stricter definition of decomposable sets by modifying condition 3) for decomposability to

$$\Re\langle S(k, m, n) \rangle = \Re\langle \bar{\mathbf{h}}^H \Phi(k, m, n) \bar{\mathbf{h}} \rangle \geq 0 \quad \forall \bar{\mathbf{h}}. \quad (26)$$

Note that this definition is, in general, less strict than the *orthogonal decomposability* condition proposed in [22] for multiuser systems in flat-fading channels, which in this case would be

$$\Re\langle S(k, m, n) \rangle = 0 \quad \forall \bar{\mathbf{h}}. \quad (27)$$

At this point, we recall the fact (see [23]) that any matrix with complex entries A can be written uniquely as $A = A_S + jA_T$, where A_S and A_T are Hermitian matrices and are given by $A_S = (A + A^H)/2$ and $A_T = -j(A - A^H)/2$. Thus, we can write

$$\Phi(k, m, n) = \Phi_S(k, m, n) + j\Phi_T(k, m, n) \quad (28)$$

where $\Phi_S(k, m, n)$ and $\Phi_T(k, m, n)$ are Hermitian. Substituting (28) in (26), we obtain a decomposability condition for single-path fading as

$$\Phi_S(k, m, n) \geq 0 \quad (29)$$

which, in other words, states that $\Phi_S(k, m, n)$ is positive semidefinite. This condition is clearly weaker than the orthogonal condition, which requires $\Phi_S(k, m, n) = \mathbf{0}$, thus allowing a larger number of patterns to be treated as decomposable. However, in the particular case of quasi-static fading, we can apply (13) and write $\Psi(k, m, n)$ as

$$\Psi(k, m, n) = \Delta \mathcal{D}_{km}^H \Delta \mathcal{D}_{kn} \odot R \quad (30)$$

where we denote $\Delta \mathcal{D}_{\alpha\beta} = \mathcal{D}_\alpha - \mathcal{D}_\beta$. From (14) and condition 2), it immediately follows that $\text{tr}[\Phi(k, m, n)] = \text{tr}[\Phi_S(k, m, n)] = 0$, and (26) becomes

$$\Phi_S(k, m, n) = \mathbf{0}. \quad (31)$$

Thus, for the quasi-static case, (26) and (27) are equivalent.

We are now ready to state channel-independent conditions for decomposability of error patterns.

Definition 2: An error pattern $\mathbf{e}_{kl} \in E_k$ is decomposable into patterns $\mathbf{e}_{km} \in E_k$ and $\mathbf{e}_{kn} \in E_k$ (denoted by $\mathbf{e}_{kl} \doteq \mathbf{e}_{km} + \mathbf{e}_{kn}$) if:

- 1*) $\mathbf{e}_{kl} = \mathbf{e}_{km} + \mathbf{e}_{kn}$;
- 2*) $[\mathbf{e}_{kl}]_{ij} = 0 \Leftrightarrow [\mathbf{e}_{km}]_{ij} = [\mathbf{e}_{kn}]_{ij} = 0$;
- 3*) $\Phi_S(k, m, n) \geq 0$.

Note that 1*) and 2*) are exactly the same as 1) and 2), respectively, but 3) has been replaced by the stronger condition 3*). The set of channel-independent indecomposable error patterns in E_k is denoted by F_k^* . Clearly, $F_k(\bar{\mathbf{h}}) \subseteq F_k^* \subseteq E_k$. By summing over F_k^* instead of $F_k(\bar{\mathbf{h}})$, we can upper-bound (25) in closed form

$$P_k \leq E_{\bar{\mathbf{h}}} \left\{ \sum_{\mathbf{e}_{kl} \in F_k^*} Q(\sqrt{2\sigma_t} \|S(k, l, l)\|) \right\} = \sum_{k=1}^{K_c} \sum_{\mathbf{e}_{k,j} \in F_k^*} \theta(k, j). \quad (32)$$

Finally, by averaging (32) over all possible transmitted codewords, we obtain a third performance index

$$\text{IUB} \triangleq \sum_{k=1}^{K_c} \sum_{\mathbf{e}_{k,j} \in F_k^*} \theta(k, j). \quad (33)$$

Note that, by construction, $IUB \leq UB$. Thus, the SER and the performance bounds can be ordered as

$$K_c \cdot SER \leq IUB \leq UB \leq CB. \quad (34)$$

We now turn our attention to some simple properties of indecomposable patterns as defined by 1*)-3*) above. These properties have practical importance, mainly in reducing the amount of computations necessary to find indecomposable error patterns or compute the union bound in (33).

C. Properties of Indecomposable Error Patterns

Property 1: For any k and l , an error pattern \mathbf{e}_{kl} with $w(\mathbf{e}_{kl}) = 1$ is always indecomposable.

Property 2: For any k and l , if an error pattern $\mathbf{e}_{kl} \notin F_k^*$, then $\mathbf{e}_{lk} \notin F_l^*$. Consequently, if $\mathbf{e}_{kl} \in F_k^*$, then $\mathbf{e}_{lk} \in F_l^*$.

The immediate practical consequence of this property is that the computational cost of searching for indecomposable sets can be cut by half, since only half of the possible error patterns needs to be tested for conditions 1*)-3*).

Proof: First we note that, for any choice of codewords x, y, z , it is always true that

$$\mathbf{e}_{xy} = -\mathbf{e}_{zx} + \mathbf{e}_{zy} \quad (35)$$

It is clear that $\mathbf{e}_{lk} \in E_l$, by definition. If $\mathbf{e}_{kl} \notin F_k^*$, then $\mathbf{e}_{kl} \doteq \mathbf{e}_{km} + \mathbf{e}_{kn}$ for some m and n . Now, to prove the property, we need to show that $\mathbf{e}_{lk} \doteq \mathbf{e}_{l\alpha} + \mathbf{e}_{l\beta}$ for some α and β . Note that the ordering of the subscripts is crucial. Observing that $\mathbf{e}_{lk} = -\mathbf{e}_{kl}$, we have that $\mathbf{e}_{lk} = -\mathbf{e}_{km} - \mathbf{e}_{kn}$; however, this is not a valid decomposition for \mathbf{e}_{lk} in the sense of 1*). Thus

$$-\mathbf{e}_{km} = -\mathbf{e}_{kl} + \mathbf{e}_{kn}. \quad (36)$$

We now observe that the right side of (36) is of the same form as the right side of the general form given in (35). Thus, comparing both expressions, it is clear that $-\mathbf{e}_{km} = -\mathbf{e}_{kl} + \mathbf{e}_{kn} = \mathbf{e}_{ln}$. Similarly, $-\mathbf{e}_{kn} = -\mathbf{e}_{kl} + \mathbf{e}_{km} = \mathbf{e}_{lm}$. Thus, we have that $\mathbf{e}_{lk} = \mathbf{e}_{ln} + \mathbf{e}_{lm}$, which is condition 1*) for decomposability of \mathbf{e}_{lk} . Conditions 2*) and 3*) are also satisfied, since $\mathbf{e}_{ln} = -\mathbf{e}_{km}$ and $\mathbf{e}_{lm} = -\mathbf{e}_{kn}$. This proves that $\mathbf{e}_{kl} \notin F_k^* \Rightarrow \mathbf{e}_{lk} \notin F_l^*$ and consequently, that $\mathbf{e}_{kl} \in F_k^* \Rightarrow \mathbf{e}_{lk} \in F_l^*$.

Property 3: F_k^* is invariant to channel or spreading codes for any k if the following three conditions are met:

- 1) fixed spreading codes are used within each block, ie, $R(n) = R\forall n$;
- 2) the channel is quasi-static;
- 3) $[R]_{ij} \neq 0 \forall i, j$.

The property allows reduction of computations in performance analysis of code sets. Once the indecomposable patterns of a code set are found, (33) can be used to compute the union bound for different R , without the need for recomputing the indecomposable sets F_k^* , $k = 1, \dots, K_c$.

Proof: Since 1*) and 2*) are independent of \mathbf{R} , we only need to check 3*). If $\mathbf{e}_{kl} \doteq \mathbf{e}_{km} + \mathbf{e}_{kn}$, we have, from 3*) and (30), that

$$\Psi(k, m, n) = \Delta \mathcal{D}_{km}^H \Delta \mathcal{D}_{kn} \odot R = \mathbf{0}. \quad (37)$$

From (37), if R does not contain zero elements, the decomposability criterion becomes

$$[\Delta \mathcal{D}_{km}^H \Delta \mathcal{D}_{kn} + \Delta \mathcal{D}_{kn}^H \Delta \mathcal{D}_{km}] = \mathbf{0} \quad (38)$$

which is independent of R , and thus proves the property.

V. THE PUB AND ITS SP APPROXIMATION

As noted earlier, the performance bounds developed so far are asymptotically tight, but fail to give an accurate performance prediction under low-SNR scenarios. This is true even for the IUB, in general, since the amount of possible expurgation in the union-bound expression varies highly between different code-sets. In this section, we first address this issue by developing a generalization of the union bound, denoted by PUB [24]. As will be shown, its major advantage over the other bounds is the facilitation of a tradeoff between its computational complexity and tightness. Its drawback, however, is that it cannot be computed in closed form, except for a very special case. Therefore, we obtain a semianalytic expression for the PUB by developing its SP approximation [25], [26].

A. PUB Derivation

Denoting by $\bar{\mathbf{y}}_i$ the matched-filter received signal corresponding to codeword D_i being transmitted, we recall that

$$\bar{\mathbf{y}}_i = \sqrt{\sigma_t} \bar{\mathbf{R}} \bar{D}_i \bar{\mathbf{h}} + \bar{\mathbf{m}}. \quad (39)$$

Therefore, the effective log-likelihood of codeword D_j if D_i is transmitted, denoted by $T_{j|i}$, is given by

$$T_{j|i} = -(\bar{\mathbf{y}}_i - \sigma_t \bar{\mathbf{R}} \bar{D}_j \bar{\mathbf{h}})^H \bar{\mathbf{R}}^{-1} (\bar{\mathbf{y}}_i - \sigma_t \bar{\mathbf{R}} \bar{D}_j \bar{\mathbf{h}}). \quad (40)$$

Using a technique similar to [24], we now determine an exact expression for the performance of an STBC employing maximum-likelihood detection. First, we define the event

$$M_{j,k|i} \triangleq \{D_j \text{ more likely than } D_k \text{ when } D_i \text{ is transmitted}\} \quad (41)$$

$$= \{T_{j|i} > T_{k|i}\} \quad (42)$$

where the right-hand side of (41) follows from (40).

Now, from (41), the probability of detecting D_j when D_i is transmitted corresponds to

$$P\{D_i \rightarrow D_j\} = P \left\{ \bigcap_{k \neq j} M_{j,k|i} \right\} = P \left\{ T_{j|i} \geq \max_{k \neq j} [T_{k|i}] \right\}. \quad (43)$$

We denote the codeword difference matrix Δ_{ij} by

$$\Delta_{ij} = \bar{D}_i - \bar{D}_j. \quad (44)$$

By substituting (39) and (40) into (41), and performing some simplifications, we can show that

$$M_{j,k|i} = \left\{ \mathbf{w}^H Q_{j,k}^{(i)} \mathbf{w} \leq 0 \right\} \triangleq \left\{ z_{j,k}^{(i)} \leq 0 \right\} \quad (45)$$

where

$$z_{j,k}^{(i)} \triangleq \mathbf{w}^H Q_{j,k}^{(i)} \mathbf{w} \quad (46)$$

$$\mathbf{w} = [\bar{\mathbf{h}}^T, \bar{\mathbf{m}}^T] \in \mathbb{C}^{L_t L_r (N_c + 1) \times 1} \quad (47)$$

$$\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{K}_w) \quad (48)$$

$$\mathbf{K}_w = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix} \in \mathbb{R}^{L_t L_r (N_c + 1) \times L_t L_r (N_c + 1)} \quad (49)$$

$$Q_{j,k}^{(i)} = \begin{bmatrix} \sigma_t \Phi(j, i, i) - \sigma_t \Phi(k, i, i) & \sqrt{\sigma_t} (\Delta_{ji}^H - \Delta_{ki}^H) \\ \sqrt{\sigma_t} (\Delta_{ji} - \Delta_{ki}) & \mathbf{0} \end{bmatrix} \quad (50)$$

where $Q_{j,k}^{(i)} \in \mathbb{C}^{L_t L_r (N_c + 1) \times L_t L_r (N_c + 1)}$. Thus, $\mathbf{z}_j^{(i)} \triangleq [z_{j,1}^{(i)}, z_{j,2}^{(i)}, \dots, z_{j,j-1}^{(i)}, z_{j,j+1}^{(i)}, \dots, z_{j,K_c}^{(i)}]^T$ is a $(K_c - 1) \times 1$ vector of complex Gaussian quadratic forms consisting of a sufficient set of metrics for the exact determination of $P\{D_i \rightarrow D_j\}$. Its probability density function (pdf) is given by

$$f(\mathbf{z}_j^{(i)}) = \frac{1}{(2\pi j)^{K_c - 1}} \int_{\mathcal{C}} \mu_z(\Delta^j) \exp[-(\Delta^j)^T \mathbf{z}_j^{(i)}] d\Delta^j \quad (51)$$

where we denote $\Delta^j \triangleq [\lambda_1 \cdots \lambda_{j-1}, \lambda_{j+1} \cdots \lambda_{K_c}]$ and $\mu_z(\Delta^j)$ is the moment generating function of $\mathbf{z}_j^{(i)}$ and is given by [25]

$$\mu_z(\Delta^j) = \frac{1}{\det[\mathbf{I} - \sum_{m=1, m \neq j}^{K_c} \lambda_m C_m]} \quad (52)$$

$$\triangleq \frac{1}{\det[\mathbf{I} - D(\Delta^j)]} \triangleq \frac{1}{\det P(\Delta^j)} \quad (53)$$

where $C_m \triangleq S Q_{j,m}^{(i)} S^H$ and $S^H S = K_w$. We also define $D(\Delta^j) \triangleq \sum_{m=1, m \neq j}^{K_c} \lambda_m C_m$ and $P(\Delta^j) \triangleq \mathbf{I} - D(\Delta^j)$. The region of analyticity (ROA) of $\mu_z(\Delta^j)$ is given by

$$\text{ROA}(\mu_z) = \{\Delta^j | \text{largest eigenvalue of } D(\Delta^j) \text{ is } < 1\}. \quad (54)$$

Using (45), $P\{D_i \rightarrow D_j\}$ can be expressed as

$$P\{D_i \rightarrow D_j\} = P\{\mathbf{z}_j^{(i)} \leq \mathbf{0}_{(K_c - 1) \times 1}\}. \quad (55)$$

We compute the probability of this error event by integrating the pdf in (51)

$$P\{D_i \rightarrow D_j\} = \int_{-\infty}^0 \cdots \int_{-\infty}^0 f(\mathbf{z}_j^{(i)}) d\mathbf{z}_j^{(i)} \quad (56)$$

where $(K_c - 1)$ integrals have to be performed. By substituting (51) and (52) into (56) and switching the integration order, we achieve

$$P\{D_i \rightarrow D_j\} = \frac{1}{(2\pi j)^{K_c - 1}} \int_{\mathcal{C}} \frac{\mu_z(\Delta^j)}{\prod_{m=1, m \neq j}^{K_c} (-\lambda_m)} d\Delta^j \quad (57)$$

with the integration contour chosen so that

$$\mathcal{C} : \{\Re(\lambda_m) < 0 \forall m\} \cap \text{ROA}(\mu_z). \quad (58)$$

The evaluation of the event probability in (43) given by the integral in (57) is analytically unsolvable, due to the number of necessary metric comparisons. If however, only one comparison is performed, the error probability can be bounded via an exact expression. This bound is obtained from (43) as

$$P\{D_i \rightarrow D_j\} = P\left\{ \bigcap_{k \neq j} M_{j,k|i} \right\} \leq P_1\{D_i \rightarrow D_j\} \quad (59)$$

$$\triangleq P\{M_{j,i|i}\} = P\{T_{j|i} \geq T_{i|i}\} \quad (60)$$

where the subscript in $P_1\{D_i \rightarrow D_j\}$ indicates that only one metric comparison is performed. Not surprisingly, this yields the expression for the PEP given in (15).

While (43) gives the exact error probability, (59) considers only the event $M_{j,k|i}$ for $k = i$, and is a common method for bounding this probability of error. The classical union bound for the symbol-error probability (SEP) given in (16) can thus be rewritten as

$$\text{SER} \leq \frac{1}{K_c} \text{UB} = \frac{1}{K_c} \sum_{i=1}^{K_c} \sum_{j=1, j \neq i}^{K_c} P_1\{D_i \rightarrow D_j\} \quad (61)$$

while the exact expression is

$$\text{SER} = \frac{1}{K_c} \sum_{i=1}^{K_c} \sum_{j=1, j \neq i}^{K_c} P\{D_i \rightarrow D_j\}. \quad (62)$$

A bound that is tighter than (61) can be obtained by performing more than one metric comparison in (59). To achieve a compromise between computational complexity and tightness, we consider M comparisons ($M > 1$) instead of all, as in (62), or only one, as in (61). For instance, if $M = 2$, we modify (59) as

$$P\{D_i \rightarrow D_j\} \leq P_2\{D_i \rightarrow D_j\} \triangleq P\{M_{j,i|i} \cap M_{j,l|i}\} \quad (63)$$

which performs two comparisons, since a third codeword D_l is taken into consideration. Clearly, the choice of D_l impacts the tightness of the resulting bound (although $P_2 \leq P_1$ always). Therefore, we use the PEP expression in (15) to select the codeword $D_l (l \neq j)$ which is most likely to have the highest impact on the bound. This criterion can be stated as

$$l = \arg \max_{\alpha} \theta(i, \alpha). \quad (64)$$

Denoting by $P_M\{D_i \rightarrow D_j\}$ the resulting error-probability bound for M metric comparisons, we can define the PUB for M comparisons, PUB_M , as

$$K_c \cdot \text{SER} \leq \text{PUB}_M \triangleq \sum_{i=1}^{K_c} \sum_{j=1, j \neq i}^{K_c} P_M\{D_i \rightarrow D_j\}. \quad (65)$$

Although (62) and (65) give us, respectively, the exact SEP and its PUB, they cannot be computed in closed form, since $P\{D_i \rightarrow D_j\}$ and $P_M\{D_i \rightarrow D_j\}$ cannot be analytically computed. In [24], a multidimensional numerical integration was proposed to compute bounds of a similar form. Computationally, however, this is a highly nontrivial task, especially when the codesets and/or block sizes become larger. In this paper, we employ the SP technique presented in [25] and [26] to obtain a semiclosed-form expression which can approximate (65) closely, even at low-SNR values, and without the need to perform a numerical integration. The next section describes this technique.

B. SP Approximation

For notational simplicity, we derive the SP approximation for the exact expression in (57). Its extension to the progressive error probability is straightforward. Our approach for obtaining the SP approximation extends the method described in [25] and [26].

The SP approximation consists of first determining a real SP for the integrand expression in (57), then a Taylor series expansion of the integrand is carried out around the SP. By truncating this expansion at the second-order term, the integration can be performed analytically and a closed-form approximation is obtained.

We start by rewriting (57) as

$$P(D_i \rightarrow D_j) = \frac{1}{(2\pi j)^{K_c-1}} \int_{\mathcal{C}} \exp[\Lambda(\underline{\lambda}^j)] d\underline{\lambda}^j \quad (66)$$

where

$$\Lambda(\underline{\lambda}^j) = -\log(\det P(\underline{\lambda}^j)) - \sum_{m=1, m \neq j}^{K_c} \log(-\lambda_m). \quad (67)$$

The real SP $\hat{\underline{\lambda}}$ has to satisfy the simultaneous equations

$$\left[\frac{\partial \Lambda(\underline{\lambda}^j)}{\partial \lambda_m} \right]_{\hat{\underline{\lambda}}} = 0, \quad \text{for } m = 1, \dots, K_c, \quad m \neq j. \quad (68)$$

Therefore, each integration contour in \mathcal{C} is taken to pass through this real SP, which must lie in the region specified by (58). A multidimensional search technique, such as the Newton-Raphson search, may be used to find the SP $\hat{\underline{\lambda}}$. Once it is found, we expand $\Lambda(\underline{\lambda}^j)$ around the SP $\hat{\underline{\lambda}}$

$$\begin{aligned} \Lambda(\underline{\lambda}^j) &= \Lambda(\hat{\underline{\lambda}}) + \sum_{m=1, m \neq j}^{K_c} \left[\frac{\partial \Lambda(\underline{\lambda}^j)}{\partial \lambda_m} \right]_{\hat{\underline{\lambda}}} (\lambda_m - \hat{\lambda}_m) \\ &+ \sum_{m=1, m \neq j}^{K_c} \sum_{n=1, n \neq j}^{K_c} \frac{\Lambda_2(m, n) (\lambda_m - \hat{\lambda}_m)^2}{2} + \dots \quad (69) \end{aligned}$$

where

$$\Lambda_2(m, n) = \left[\frac{\partial^2 \Lambda(\underline{\lambda}^j)}{\partial \lambda_m \partial \lambda_n} \right]_{\hat{\underline{\lambda}}}, \quad m, n = 1, \dots, K_c \text{ and } m, n \neq j. \quad (70)$$

The $(K_c - 1) \times (K_c - 1)$ Hessian matrix Λ_2 at $\hat{\underline{\lambda}}$ is

$$\Lambda_2 \triangleq [\Lambda_2(m, n)], \quad m, n = 1, \dots, K_c \text{ and } m, n \neq j. \quad (71)$$

By truncating (69) at the second order, and substituting (68) in (66), we have

$$\begin{aligned} P(D_i \rightarrow D_j) &\approx \frac{1}{(2\pi)^{K_c-1}} \int \exp \left[\Lambda(\hat{\underline{\lambda}}) \right. \\ &\left. + \sum_{m,n=1, m,n \neq j}^{K_c} \frac{\Lambda_2(m, n) (\lambda_m - \hat{\lambda}_m)^2}{2} \right] d\underline{\lambda}^j. \quad (72) \end{aligned}$$

By performing the change of variables

$$\lambda_m = \hat{\lambda}_m + j s_m \quad m = 1, \dots, K_c \text{ and } m \neq j \quad (73)$$

and defining $\underline{s}^j = [s_1 \cdots s_{j-1}, s_{j+1} \cdots s_{K_c}]$ we have

$$\begin{aligned} P(D_i \rightarrow D_j) &\approx \frac{\exp[\Lambda(\hat{\underline{\lambda}})]}{(2\pi)^{K_c-1}} \\ &\times \int_{-\infty}^{+\infty} \exp \left[-\frac{1}{2} \sum_{m,n} \Lambda_2(m, n) s_m s_n \right] d\underline{s} \quad (74) \end{aligned}$$

which finally yields

$$P(D_i \rightarrow D_j)^{(0)} \approx \frac{\exp[\Lambda(\hat{\underline{\lambda}})]}{(2\pi)^{(K_c-1)/2} [\det(\Lambda_2)]^{1/2}} \quad (75)$$

which is known as the zeroth-order approximation to the integral in (57). Since Λ_2 is always positive definite in our problem, the approximation is guaranteed to always yield a positive value [25], [26].

A first-order approximation is obtained by

$$P\{D_i \rightarrow D_j\}^{(1)} = e^{c_t} P(D_i \rightarrow D_j)^{(0)} \quad (76)$$

$$= \frac{\exp[c_t \Lambda(\hat{\underline{\lambda}})]}{(2\pi)^{(K_c-1)/2} [\det(\Lambda_2)]^{1/2}} \quad (77)$$

where c_t is a correction term, which is a function of the third- and fourth-order partial derivatives of $\Lambda(\underline{\lambda}^j)$ and need only be computed after the real SP is found. Detailed expressions for c_t can be found in [25]. The final step is to write the SER and PUB approximations

$$\text{SER} \approx \frac{1}{K_c} \sum_{i=1}^{K_c} \sum_{j=1, j \neq i}^{K_c} P\{D_i \rightarrow D_j\}^{(1)} \quad (78)$$

$$\text{PUB}_M \approx \sum_{i=1}^{K_c} \sum_{j=1, j \neq i}^{K_c} P_M\{D_i \rightarrow D_j\}^{(1)}. \quad (79)$$

Fig. 1 compares the PUB approximations with the exact UB for a 2×2 quaternary (Q)PSK STBC with eight codewords (see

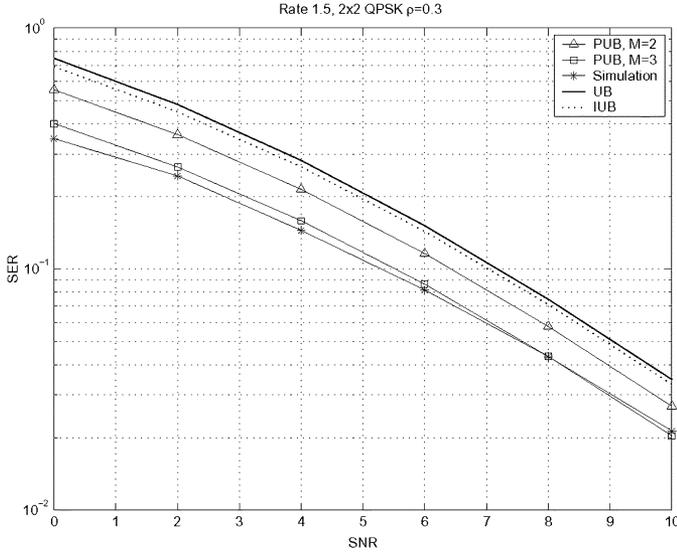


Fig. 1. UB, IUB, and PUB for rate-1.5, 2×2 , QPSK, spread system.

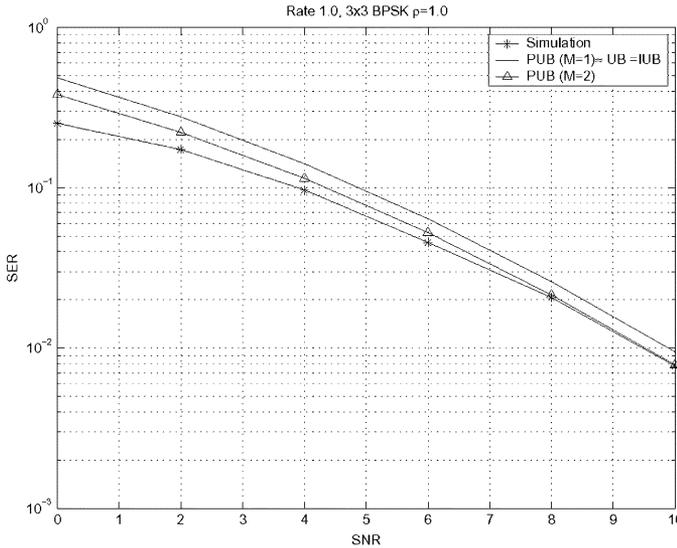


Fig. 2. UB, IUB, and PUB for rate-1.0, 3×3 , BPSK, nonspread system.

[7]). The UB and IUB for the code are also shown for comparison. Note that the PUB approximation is significantly more accurate than the IUB, which, in turn, is tighter than the UB, as expected. Since it takes more decision metrics into consideration, the PUB clearly gives a much better prediction for the code performance. Furthermore, the choice of M yields a tradeoff between approximation accuracy and numerical complexity. Fig. 2 displays similar approximations for a 3×3 binary (B)PSK code. We note that in this case, the IUB curve turns out to be exactly the same as the UB curve (due to the fact that the code contains no decomposable patterns).

VI. CODE CONSTRUCTION AND ISOMETRIES

In the previous sections, we presented several techniques (Chernoff, union, indecomposable, and PUBs) for use in evaluating the performance of an STBC system. With this performance criteria in hand, we can, in principle, search for code sets which optimize these quantities.

However, due to the complicated nature of the codeword space (nonlinear, nonmetric), constructing codeword sets that optimize any of the performance criteria described in the previous sections is a difficult task. This is especially the case for the PUB criterion, since it involves searching numerically for an SP for each pairwise error in the codeset. Thus, as these sets become large (for higher rates), not only does the search space increase, but the PUB computation for each set becomes more challenging.

Recently, in [8], a code construction method for STBCs was proposed that takes advantage of certain “distance-preserving” transformations in order to hierarchically build higher rate codes from smaller sets. These transformations are called *isometries*. An isometry is an operation ϕ over the codewords of a set \mathcal{C} which results in a new set \mathcal{C}^* , such that the “distance” (e.g., rank criterion or coding gain) between codewords is preserved. Specifically, we denote

$$\mathcal{C}^* = \phi(\mathcal{C}) \triangleq \{\phi(\mathcal{D}_i) | \mathcal{D}_i \in \mathcal{C}\}. \quad (80)$$

In [8], isometries were applied to the design of codes by optimizing a worst-case performance criteria. We instead apply isometries to PUB optimization. Therefore, ϕ must satisfy $P_M\{\mathcal{D}_i \rightarrow \mathcal{D}_j\} = P_M\{\phi(\mathcal{D}_i) \rightarrow \phi(\mathcal{D}_j)\} \forall i, j$. Denoting by \mathcal{D} any codeword in \mathcal{C} , the following operations can be shown to be valid isometries for the PUB:

- I1) $\phi(\mathcal{D}) = U\mathcal{D}$, U being a unitary matrix $U^H U = I$;
- I2) $\phi(\mathcal{D}) = \mathcal{D}P$, P being a unitary matrix $P^H P = P$.

Note that for STBCs formed via PSK constellations, U and P are required to have only one nonzero element in each row and column, and these nonzero elements should be drawn from the same PSK alphabet as \mathcal{D} . For nonspread systems, both I1) and I2) constitute isometric transformations, whereas only I1) applies to spread systems, in general [8].

It can be shown that the union-bound measures UB, CB, IUB, and PUB for \mathcal{C}^* are the same as in \mathcal{C} . Thus, isometric operations enable us to reuse a good codeword structure found for lower cardinality sets and duplicate it, consequently doubling the cardinality of the code. In order to employ the union bound to isometric code constructions for nonspread systems, we herein present a modification of the greedy algorithm proposed in [8].

1. Start with a good set \mathcal{C}_k .
2. Generate the set \mathcal{C}_k^* from \mathcal{C}_k such that

$$\mathcal{C}_k^* = \{U_m \mathcal{C}_k P_n | \min_{m,n} \text{SER}(\mathcal{C}_k \cup U_m \mathcal{C}_k P_n)\}. \quad (81)$$

3. $\mathcal{C}_{k+1} = \mathcal{C}_k \cup \mathcal{C}_k^*$. If desired rate is achieved, stop. Otherwise, go to step 2.

We use $\text{SER}(\mathcal{C})$ to denote any of the union-bound performance measures discussed before for a given codeset \mathcal{C} . If a design for spread system is desired, we simply replace (81) by

$$\mathcal{C}_k^* = \{U_m \mathcal{C}_k | \min_m \text{SER}(\mathcal{C} \cup U_m \mathcal{C})\}. \quad (82)$$

TABLE I
OPTIMAL CODES

index	ρ	rate	constl	size	OMM	CB	UB
1	1	1	2×2	BPSK	[1,7,8,14]	ditto OMM	ditto OMM
2	1	1	2×2	QPSK	[1,41,156,247]	[2,42,156,180]	ditto CB
3	1	1	2×2	8PSK	[3,1314,2167,3414]	[4,803,2224,2967]	ditto CB
4	1	1	3×3	BPSK	$\begin{bmatrix} 1 & 84 & 166 & 248 \\ 282 & 335 & 429 & 499 \end{bmatrix}$	ditto OMM	$\begin{bmatrix} 1 & 84 & 189 & 232 \\ 282 & 335 & 422 & 499 \end{bmatrix}$
5	1	1.5	2×2	QPSK	$\begin{bmatrix} 2 & 42 & 93 & 117 \\ 128 & 168 & 223 & 247 \end{bmatrix}$	ditto OMM	ditto OMM
6	1	2	2×2	QPSK	$\begin{bmatrix} 0 & 20 & 42 & 62 \\ 70 & 82 & 107 & 127 \\ 133 & 145 & 173 & 185 \\ 195 & 215 & 236 & 248 \end{bmatrix}$	$\begin{bmatrix} 2 & 30 & 42 & 54 \\ 65 & 93 & 105 & 117 \\ 128 & 156 & 168 & 180 \\ 195 & 223 & 235 & 247 \end{bmatrix}$	ditto CB
7	0.3	1	2×2	BPSK	[0,6,11,13]	ditto OMM	ditto OMM
8	0.3	1	2×2	QPSK	[1,87,174,248]	[1,88,191,230]	[1,42,156,183]
9	0.3	1	2×2	8PSK	[3,1314,2447,3246]	ditto OMM	ditto OMM
10	0.3	1	3×3	BPSK	$\begin{bmatrix} 0 & 31 & 99 & 124 \\ 421 & 442 & 454 & 473 \end{bmatrix}$	$\begin{bmatrix} 1 & 30 & 232 & 247 \\ 331 & 340 & 418 & 445 \end{bmatrix}$	$\begin{bmatrix} 10 & 87 & 180 & 233 \\ 317 & 352 & 387 & 478 \end{bmatrix}$
11	0.3	1.5	2×2	QPSK	$\begin{bmatrix} 2 & 42 & 93 & 117 \\ 128 & 168 & 223 & 247 \end{bmatrix}$	$\begin{bmatrix} 2 & 8 & 87 & 110 \\ 145 & 185 & 236 & 247 \end{bmatrix}$	$\begin{bmatrix} 1 & 47 & 84 & 122 \\ 131 & 173 & 214 & 248 \end{bmatrix}$
12	0.3	2	2×2	QPSK	$\begin{bmatrix} 0 & 10 & 37 & 47 \\ 82 & 88 & 119 & 125 \\ 133 & 143 & 160 & 170 \\ 215 & 221 & 242 & 248 \end{bmatrix}$	$\begin{bmatrix} 2 & 20 & 42 & 60 \\ 75 & 93 & 99 & 117 \\ 128 & 150 & 168 & 190 \\ 201 & 223 & 225 & 247 \end{bmatrix}$	[ditto CB]

TABLE II
DISTANCE SPECTRUM

	rule	DU	CB	(β_0, γ_0)	(β_1, γ_1)	(β_2, γ_2)	(β_3, γ_3)	(β_4, γ_4)	(β_5, γ_5)	(β_6, γ_6)
1	OMM	Y	0.2812	(2,0.0625)	(4,0.0327)					
	UB	Y	0.2812	(2,0.0625)	(4,0.0327)					
2	OMM	N	0.2903	(1,0.0625)	(4,0.0500)	(1,0.0278)				
	UB	Y	0.2361	(2,0.0625)	(4,0.0278)					
3	OMM	Y	0.2321	(4,0.0397)	(2,0.0366)					
	UB	Y	0.2189	(2,0.0467)	(2,0.0341)	(2,0.0278)				
4	OMM	N	0.3672	(22,0.0156)	(6,0.0039)					
	UB	N	0.3672	(22,0.0156)	(6,0.0039)					
5	OMM	Y	1.5625	(24,0.0625)	(4,0.0156)					
	UB	Y	1.5625	(24,0.0625)	(4,0.0156)					
6	OMM	N	14.6285	(24,0.2500)	(52,0.1250)	(26,0.0625)	(8,0.0312)	(8,0.0278)	(2,0.0156)	
	UB	Y	12.0139	(32,0.2500)	(48,0.0625)	(32,0.0278)	(8,0.0156)			
7	OMM	Y	0.2259	(4,0.0625)	(2,0.0156)					
	UB	Y	0.2259	(4,0.0625)	(2,0.0156)					
8	OMM	N	0.2278	(4,0.0423)	(1,0.0305)	(1,0.0281)				
	UB	N	0.2272	(4,0.0423)	(1,0.0302)	(1,0.0278)				
9	OMM	Y	0.2151	(4,0.0366)	(2,0.0343)					
	UB	Y	0.2151	(4,0.0366)	(2,0.0343)					
10	OMM	N	0.1309	(4,0.0082)	(4,0.0055)	(4,0.0046)	(8,0.0043)	(4,0.0029)	(4,0.0028)	
	UB	N	0.1271	(4,0.0082)	(4,0.0055)	(4,0.0043)	(4,0.0042)	(4,0.0039)	(4,0.0029)	(4,0.0028)
11	OMM	Y	1.5625	(24,0.0625)	(4,0.0156)					
	UB	N	1.5359	(2,0.0687)	(8,0.0654)	(10, 0.0625)	(8,0.0312)			
12	OMM	N	8.8273	(64,0.0859)	(32,0.0687)	(16,0.0625)	(4,0.0156)	(4,0.0172)		
	UB	Y	8.7988	(64,0.0887)	(48,0.0625)	(8,0.0156)				

Although suboptimal, in general, designs obtained via isometries have been shown to yield very good codes, sometimes in exact agreement with the optimal codes obtained via full exhaustive search [8].

VII. CODE SEARCH RESULTS

Our results are divided in two parts. First, we present exhaustive search results for small cardinality codes that optimize UB and CB. Due to its higher complexity, we do not consider an exhaustive search using the PUB criterion. Our results are compared with the OMM codes [6] that optimize the worst-case PEP. Searches using UB and PUB are performed assuming that SNR = 1 dB. It is important to stress that all the performance criteria developed in this paper take into consideration the entire distance spectrum of the codes, whereas the classical approach

of rank and determinant maximization only accounts for the worst-case error probability scenario. We characterize the distance spectrum of a code with cardinality K by a matrix Υ with entries $\Upsilon_{ij} = \theta(i, j)$ or $\Upsilon_{ij} = |\Phi(i, j, j)|$, with $1 \leq i, j \leq K_c$.

In the second part, we present search results employing a hierarchical construction via isometries by optimizing the PUB. Throughout this section, we consider OMM codes as the baseline for comparison against new found codes. These are codes that maximize the performance index

$$\Upsilon_{\text{OMM}} \triangleq \min_{i,j,i \neq j} |\Phi(i, j, j)|, \quad 1 \leq i, j \leq K_c. \quad (83)$$

Comparing the performance against OMM codes is justified, since they are already optimized (in the sense of worst-case

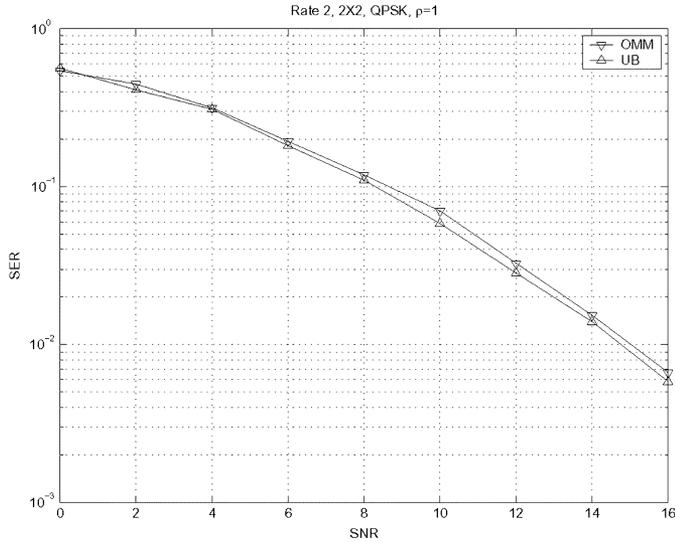


Fig. 3. Rate 2, QPSK, 2×2 , $\rho = 1.0$ union-bound code versus worst-case code.

TABLE III
CB CODES VERSUS UB CODES

	design	γ_0	UB
8	CB	0.0423	0.8031
	UB	0.0430	0.8028
11	CB	0.0687	4.2395
	UB	0.0654	4.2268
10	CB	0.0082	2.8080
	UB	0.0086	2.8047
4	CB	0.0156	3.0522
	UB	0.0156	3.0473

PEP), and have already been shown to perform better than other classical designs, such as orthogonal and unitary group codes [6].

A. Exhaustive Search

We denote the OMM, UB, and CB optimized codes by C_{OMM} , C_{UB} , and C_{CB} , respectively. Following the arguments of [6] and [7], we assume spreading code sets which are equicorrelated, which enables the modification of the correlation in a controlled manner. For spread systems, a reasonable correlation value of $\rho = 0.3$ is used, for nonspread systems $\rho = 1$. We use the term “distance uniform” (DU) to denote when the distance spectrum of a code set is symmetric such that from each codeword point of view, the “distance” ($|\Phi|$ or θ) distribution of all other codewords are identical. For ease of representation, a unique integer number, the code index, is used to represent a codeword. If n PSK is employed, u is the n th root of unity $u = e^{j(2\pi/n)}$, then $d_i(t)$ is a power of u , say $d_i(t) = u^{k_i(t)}$, thus a block code matrix of size $N_c \times L_t$ can be represented by the index

$$k_{L_t}(N_c) + k_{L_t-1}(N_c)n + \dots + k_2(1)n^{N_c L_t - 2} + k_1(1)n^{N_c L_t - 1}. \quad (84)$$

The found codes are listed in Table I. For each found set, we sort $1/|\Phi(i, j, j)|$, $i \neq j$ in descending order, denoting them accordingly by $\{\gamma_0, \gamma_1, \dots\}$, and the number of codeword pairs

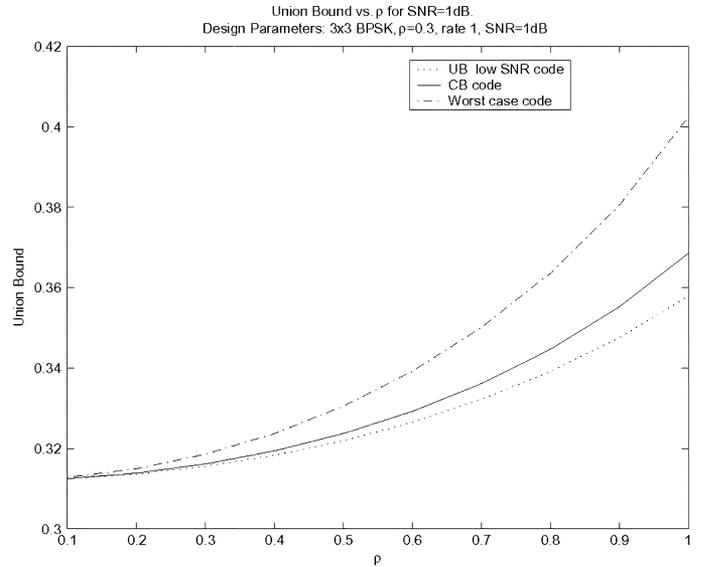


Fig. 4. Rate 1.0, 3×3 , BPSK, spread system $\rho = 0.3$, sensitivity to ρ .

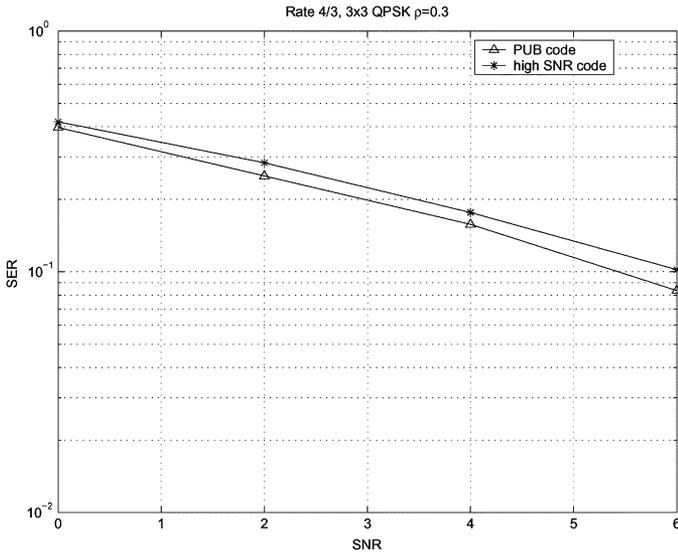
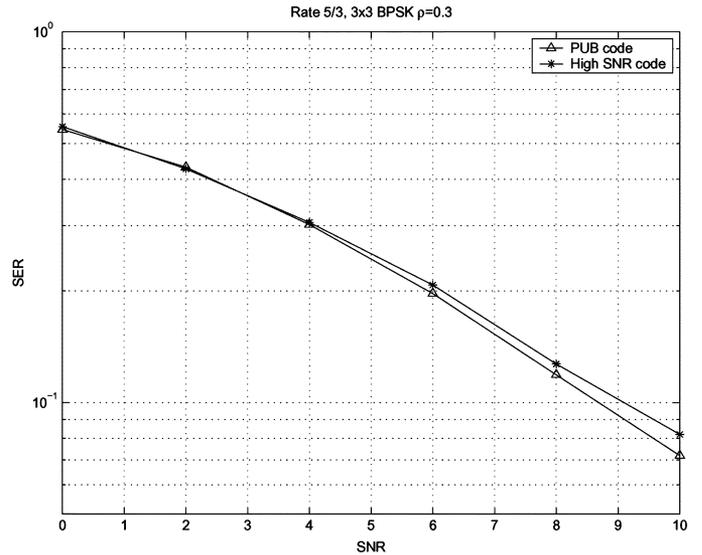
achieving γ_k by $\{\beta_0, \beta_1, \dots\}$. Clearly, $\text{CB} = \sum_k \beta_k \gamma_k$. We use (β_k, γ_k) as a shorthand note to represent β_k pairs of codewords achieving γ_k . This yields another description of the codes, in terms of their distance spectra, which are listed in Table II. For rate-1 2×2 codes (1,2,3,7,8,9 in Table I), each set has only four codewords, thus C_{CB} is either identical or slightly better than C_{OMM} for both spread and nonspread systems. For rate-1 3×3 BPSK codes (4,10), each set has eight codewords. The CB search yields slightly better code sets for spread systems, but the identical code set for nonspread systems. For rate-1.5 2×2 QPSK codes (5,11), C_{CB} is slightly better than C_{OMM} for spread systems, but identical for nonspread systems. Interestingly, the worst-case approach yields the same code set for both spread and nonspread systems, and this code is indeed the **orthogonal code** with a uniform distance spectrum [27]. Therefore, the performance of this code is independent of the spreading code correlation ρ (due to orthogonality); further search verifies that this code is optimal for all ρ by the worst-case criterion. For rate-2 2×2 QPSK codes (6,12), for nonspread systems, C_{CB} exhibits larger gains over C_{OMM} , they both have $\gamma_0 = 0.25$ and $\beta_0 = 32$ for C_{CB} , $\beta_0 = 26$ for C_{OMM} . Despite the disadvantage of β_0 , the simulation results in Fig. 3 confirm that C_{CB} has about a 0.4 dB gain over C_{OMM} . This illustrates the fact that a worst-case analysis fails to give a complete description of performance by not taking the entire distance spectrum into consideration. For spread systems, C_{CB} shows a slight gain over C_{OMM} . An inspection of the distance spectrum reveals that C_{CB} 's are DU for both spread and nonspread systems.

Results for code searches based on the UB criterion are provided in Table III. The CB codes are also shown for comparison. The UB codes yield slightly better performance. It is interesting to note that the UB code (11) for rate-1.5 QPSK 2×2 is DU, while the CB code is not. This is also true for spread and nonspread rate-1 BPSK 3×3 codes (4,10).

The UB codes also appear to be more robust to changes in the correlation value ρ , i.e., they perform better under different

TABLE IV
HIERARCHICAL CODES

index	rate	size	constl	PUB				OMM/UB			
				10	501	13	498	10	501	13	498
1	4/3	3×3	BPSK	134	377	326	185	122	389	125	386
				87	424	111	400	50	461	53	458
				53	458	50	461	66	445	69	442
				10	501	13	498	10	501	13	498
2	5/3	3×3	BPSK	134	377	326	185	122	389	125	386
				87	424	111	400	50	461	53	458
				53	458	50	461	66	445	69	442
				69	442	66	445	17	494	41	470
				17	494	41	470	87	424	111	400
				136	375	328	183	22	489	46	465
				386	125	389	122	80	431	104	407
				4	2336	292	2048	4	2336	292	2048
3	5/2	2×2	8PSK	1202	3478	1426	3254	1202	3478	1426	3254
				607	2939	895	2651	607	2939	895	2651
				1741	4073	2029	3785	1741	4073	2029	3785
				2633	877	589	2921	22	2354	310	2066
				2011	3839	4063	1787	1156	3488	1444	3200
				3236	1408	1184	3460	617	841	1759	2047
				2102	274	50	2326	2669	2893	3803	4091

Fig. 5. Rate-4/3, 3×3 , BPSK, $\rho = 0.3$, PUB code versus OMM/UB code.Fig. 6. Rate-5/3, 3×3 , BPSK, $\rho = 0.3$, PUB code versus OMM/UB code.

values of correlation. This is illustrated in Fig. 4, which shows the performance of codes in (10). For code group (4), simulation results confirm the advantage of UB codes versus CB codes at SNR of 1 dB.

B. Hierarchical Design

Exhaustive searches for codes optimizing PUB are computationally infeasible, due to the number of operations required to compute the SP approximation. Therefore, we employed a hierarchical approach to design a few sporadic codes. Another important application of hierarchical searches is the design of large cardinality code sets, since the computational cost for exhaustive search is naturally very high in this case.

Our PUB-optimized code search results are summarized in Table IV. A spread system with ($\rho = 0.3$) is assumed in all cases. It turns out that the hierarchical construction based on OMM and UB (or IUB) criteria yields the same codes, and these are also shown in Table IV.

Figs. 5–7 compare the performance of PUB and OMM/UB codes of different block sizes, cardinality, and rates. An improvement of around 0.5 dB can be observed. A closer inspection of the codes reveals that the OMM codes are DU with respect to the PUB distance measure. On the other hand, the optimal PUB codes are not. This illustrates that although distance uniformity is characteristic of many “good” codes, enforcing this property might, in some cases, entail loss in performance.

VIII. CONCLUSION

In this paper, we developed several indices for performance assessment of STBCs. The Chernoff-based and exact PEP union bounds were obtained by simple averaging of the PEPs of the set. Subsequently, it was shown that some terms in the union-bound summation were redundant, and therefore could be expurgated. Further analysis of decomposable error patterns allowed us to obtain a tighter version of the union bound, the IUB. All these bounds were revealed to still be quite loose at

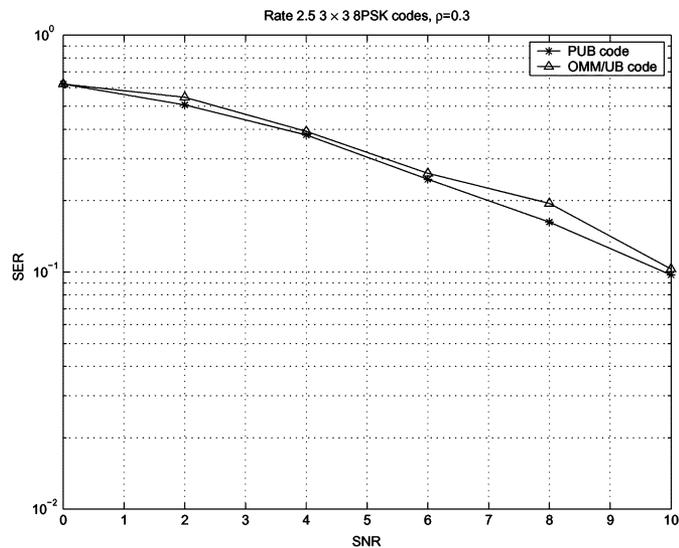


Fig. 7. Rate-5/2, 2x2, 8PSK, $\rho = 0.3$, PUB code versus OMM/UB code.

low SNR, and therefore, we also proposed the progressive union bound (PUB) as a performance index for STBCs. A semianalytic approximation for it was derived by applying a saddlepoint technique and shown to match the simulated code performance more closely than the other bounds. As another advantage, it was noted that the PUB allows a tradeoff between numerical complexity and approximation accuracy. Finally, we showed that code searches performed by optimizing the new criteria can show significant improvement over worst-case designs. Our results also indicate that optimizing tighter bounds during the searches can yield better codes, in general.

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